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Research Article

Global exponential stability of neutral high-order stochastic Hopfield neural networks with Markovian jump parameters and mixed time delays

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ABSTRACT

In this paper, a class of neutral high-order stochastic Hopfield neural networks with Markovian jump parameters and mixed time delays is investigated. The jumping parameters are modeled as a continuous-time finite-state Markov chain. At first, the existence of equilibrium point for the addressed neural networks is studied. By utilizing the Lyapunov stability theory, stochastic analysis theory and linear matrix inequality (LMI) technique, new delay-dependent stability criteria are presented in terms of linear matrix inequalities to guarantee the neural networks to be globally exponentially stable in the mean square. Numerical simulations are carried out to illustrate the main results.

1. Introduction

The human brain is made up of a large number of cells called neurons and their interconnections. An artificial neural network is an information processing system that has certain characteristics in common with biological neural networks. Since the pioneering work in [16,17], Hopfield type neural networks have been intensively studied in the past three decades and have been applied to signal and image processing, pattern recognition, fault diagnosis, optimization problems and special problems of A/D converter design, see [9,11,13,19,20,29,31,32,34,38,43] and references therein. However such neural networks are shown to have limitations such as limited capacity when used in pattern recognition problems (see, e.g., [20]). As well, the cases of optimization problems that can be solved using neural networks are limited. This led many investigators to use neural networks with high order connections. High-order neural networks allow high-order interactions between neurons, and therefore have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than the traditional first-order neural networks [35]. Recently, there has been considerable attention in the literature on high-order Hopfield type neural networks (see, e.g., [25,40–42] and the references therein). On the other hand, due to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions [28]. However, the stability analysis of neural networks of neutral-type has been investigated by only a few researchers [10,23,27,28,46].

Markovian jump systems introduced by Krasovskii and Lidskii [21] are the hybrid systems with two components in the state. The first one refers to the model which is described by a continuous-time finite-state Markovian process, and the second one refers to the state which is represented by a system of differential equations. The jump systems have the advantage of modeling the dynamic systems subject to abrupt variation in their structures, such as component failures or repairs, sudden environmental disturbance, changing subsystem interconnections, operating in different points of a nonlinear plant [26]. It should be pointed out that such a jump system has seldom been applied to neural networks due to the difficulty of mathematics. However, in real life, neural networks often exhibit information latching. It is recognized that a way of dealing with this information latching

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problem is to extract finite-state representations (also called modes or clusters). In fact, such a neural network with information latching may have finite modes, and the modes may switch (or jump) from one to another at different times, and the switching (or jumping) between two arbitrarily different modes can be governed by a Markov chain. Hence, the neural network with Markovian jump parameters has been a subject of great significance in modeling a class of neural networks with finite modes [48]. Therefore, neural networks with Markovian jump parameters have received a great deal of attention. For instance, Balasubramaniam et al. [2] investigated state estimation problem for a class of neural networks with Markovian jumping parameters, Zhang and Wang [44] discussed the problem of global asymptotical stabilization for a class of Markovian jumping stochastic Cohen–Grossberg neural networks with mixed delays including discrete delays and distributed delays, Zhu and Cao [47] studied the exponential stability problem for a class of Markovian jump impulsive stochastic Cohen–Grossberg neural networks with mixed time delays and known or unknown parameters, and so on. For details concerning stability analysis of neural networks with Markovian jump parameters, please see [3,4,6,24,49], and the references cited therein.

In addition, noise disturbance is a major source of instability and can lead to poor performances in neural networks. In real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that a neural network could be stabilized or destabilized by certain stochastic inputs [7]. Hence, great attention has been paid on the stability analysis for stochastic neural networks, and some initial results have been obtained (see, e.g., [18,33,36,37,45] and the references therein).

In addition to the noise disturbance, time delay is also a major source for causing instability and poor performances in neural networks (see, e.g., [1,5,8]). It is well-known that there exist time delays in the information processing of neurons due to various reasons. For example, time delays can be caused by the finite switching speed of amplifier circuits in neural networks or deliberately introduced to achieve tasks of dealing with motion-related problems, such as moving image processing. Time delays in the neural networks make the dynamic behaviors become more complicated, and may destabilize the stable equilibria and admit periodic oscillation, bifurcation and chaos. Therefore, considerable attention has been paid on the study of delay systems in control theory and a large body of work has been reported in the literature (see, e.g., [7,22,30,36,37] and the references therein).

To the best of our knowledge, the global exponential stability analysis problem for neutral high-order stochastic Hopfield neural networks with Markovian jump parameters and mixed time delays have not been studied thoroughly in the literature and it is very important in both theories and applications, so there exists open room for further improvement. This situation motivates our present investigation. This paper is concerned with the global exponential stability analysis problem for a class of neutral high-order stochastic Hopfield neural networks with both Markovian jump parameters and mixed time delays, which synchronously comprise constant, time-varying, and distributed delays. By utilizing the Lyapunov stability theory, stochastic analysis theory, and linear matrix inequality (LMI) technique, some novel delay-dependent conditions are obtained, which guarantee the exponential stability of the equilibrium point. The proposed LMI-based criteria are computationally efficient as they can be easily checked by using recently developed standard algorithms, such as interior-point methods [8], in solving LMIs. Finally, numerical simulations are provided to illustrate the effectiveness of the theoretical results.

The organization of this paper is as follows: in Section 2, we propose the relating notations, definitions and lemmas which would be used later, moreover, the existence of equilibrium point is proved; in Section 3, new delay-dependent global exponential stability criteria will be established for neutral high-order stochastic Hopfield neural networks with Markovian jump parameters and mixed time delays to be globally exponentially stable in the mean square; numerical simulations will be given in Section 4 to demonstrate the effectiveness of our results. Finally, conclusions are drawn in Section 5.

Notation: Throughout this paper, $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively; the superscript "T" denotes matrix transposition and the notation $X\geq Y$ (respectively, $X>Y$) where X and Y are symmetric matrices, means that $X-Y$ is semi-positive definite (respectively, positive definite); $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a real symmetric matrix, respectively; I_n is the $n\times n$ identity matrix; the notation $C^{2,1}(\mathbf{R}^+\times\mathbf{R}^n\times S;\mathbf{R}^+)$ denotes the family of all nonnegative functions $V(t,x(t),i)$ on $\mathbf{R}^+\times\mathbf{R}^n\times S$ which are continuously twice differentiable in x and once differentiable in t; $(\Omega,\mathcal{F},\mathcal{P})$ is a complete probability space, where Ω is the sample space, $\mathcal{F}$ is the $\sigma-$algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$; $L^2_{\mathcal{F}_0}([-\tau,0];\mathbf{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable $C([-\tau,0];\mathbf{R}^n)-$valued random variables $\xi=\{\xi(\theta):-\tau\leq\theta\leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0}\mathbb{E}|\xi(\theta)|^2<\infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$; the shorthand diag$\{\cdots\}$ denotes the block diagonal matrix; $|\cdot|$ is the Euclidean norm in $\mathbf{R}^n$; I is the identity matrix of appropriate dimension; and the symmetric terms in asymmetric matrix are denoted by $*$.

2. Problem formulation

In this paper, the neutral high-order stochastic Hopfield neural networks with mixed time delays is described by the following differential equation:

$$\begin{aligned} \mathrm{d}y_i(t)-k_i\,\mathrm{d}y_i(t-h) = \Bigg\{ & -c_iy_i(t)+\sum_{j=1}^{n}a_{ij}g_j(y_j(t))+\sum_{j=1}^{n}b_{ij}g_j(y_j(t-\tau_j(t))) \\ & +\sum_{j=1}^{n}\sum_{l=1}^{n}T_{ijl}g_j(y_j(t-\tau_j(t)))g_l(y_l(t-\tau_l(t))) \\ & +\sum_{j=1}^{n}d_{ij}\int_{t-\tau_j(t)}^{t}g_j(y_j(s))\,\mathrm{d}s+J_i\Bigg\}\,\mathrm{d}t \\ & +\rho_i(y_i(t),y_i(t-\tau_i(t)),t)\,\mathrm{d}\omega_i(t), \end{aligned} \tag{2.1}$$

where $i\in\{1,2,\ldots,n\}$, $t\geq t_0$; $y_i(t)$ is the neural state of cell i at time t; $c_i>0$ denotes the rate with which the cell i resets its potential to the resting state; a_{ij}, b_{ij} and d_{ij} are the first-order synaptic weights of the neural networks; T_{ijl} is the second-order synaptic weights of the neural networks; $\tau_j(t)$ $(j=1,2,\ldots,n)$ is the transmission delays of the ith unit along the axon of the jth unit at time t such that $0<\tau_j(t)\leq\overline{\tau}$ and $\dot{\tau}_j(t)\leq\varrho<1$, where $\overline{\tau}$ and ϱ are constants; the activation function g_j is continuous on $\mathbf{R}$; J_i is the external input. In general, the current state of a neuron may depend on several events having occurred in the neighboring neurons at different times. Therefore, time delays can easily be introduced in the neural network models by writing the first order interaction term in the form $\sum_{j=1}^{n}\sum_{m=1}^{l}T_{ijm}g_j(y_j(t-\tau_{ijm}))$, where m is an index that refers to past synaptic events and $\tau_{ijm}\geq 0$. k_i is the neutral delayed strength of connectivity; $h>0$ is a neutral constant delay; ρ_i is the diffusion coefficient (or noise intensity) and the stochastic disturbance $\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_n(t)]^T\in\mathbf{R}^n$ is a Brownian motion