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Parameter estimation for a dual-rate system with time delay ☆

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ABSTRACT

This paper investigates the parameter estimation problem of the dual-rate system with time delay. The slow-rate model of the dual-rate system with time delay is derived by using the discretization technique. The parameters and states of the system are simultaneously estimated. The states are estimated by using the Kalman filter, and the parameters are estimated based on the stochastic gradient algorithm or the recursive least squares algorithm. When concerning state estimate of the dual-rate system with time delay, the state augmentation method is employed with lower computational load than that of the conventional one. Simulation examples and an experimental study are given to illustrate the proposed algorithm.

1. Introduction

In many industrial processes, modeling the process system is a fundamental problem for control applications, such as soft sensor and controller design [1–4]. The main obstacle of modeling the industrial process system is the lack of the measurable input–output data due to time delay and different sampling rates between the regularly sampled data in sensors and laboratory analyses [5,6]. As the input and the output are sampled in two different sampling periods, the system is often described by a dual-rate system. The challenge of modeling is how to deal with the parameter estimation problem based on the dual-rate input and output data with time delay.

The methods for the parameter estimation of the multirate system have been widely developed. For the general multi-input, multi-output and multirate system, the system is divided into subsystems and then a least square method is used to estimate the system parameters [7]. For a fast input slow output dual-rate system, the polynomial transformation technique adopted to transform the dual-rate system into the one which can be identified by the measurable data, and the parameter and inter-sample output can be estimated by a stochastic gradient (SG) algorithm [8] and a recursive least square (RLS) algorithm [9]. For a multi-input multirate system, the discretization technique employed to convert the system into a slow-rate system which can be directly identified based on the known data and the RLS algorithm is used to estimate the system parameters [10]. A common strategy of these methods is to convert the original multirate system into systems which are identifiable based on the traditional system identification methods. For a system with very slow output samples, the output error method has been used to directly identify the fast rate model from the known fast rate input and the slow rate output [11]. Some researchers have applied the expectation maximization method to deal with the parameter estimation problem of the system with missing data. The state-space model identification based on the expectation maximization (EM) algorithm and the Kalman filtering estimation method has been adopted to identify a chemical process system based on the irregular sampled output [12]. For example, the grey-box identification techniques have been applied to the bleaching operation in a pulp mill to identify a dynamic model with irregular outputs, and the EM algorithm in the sense of maximum likelihood estimation is used to estimate the system parameters [13].

Currently, for the single rate time delay systems, many identification methods exist [14–16]; however, these methods cannot be directly applied to multirate systems. This paper focuses on the parameter estimation problem for a single-input single-output dual-rate system with time delay, which has two different sampling periods. First, the discretization technique is used to derive

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the slow-rate state-space model of the original dual-rate system which can be identified from the measurable input–output data. Second, inspired by [17–19], the observability canonical form of the obtained state-space model is derived. With some algebraic manipulation the model for the dual-rate system is obtained of which the information vector contains the unmeasurable states. Finally, the Kalman filtering algorithm is applied to estimate the model states, and the SG algorithm or the RLS algorithm is used to identify the model parameters. The parameters and states of the system are simultaneously estimated. The proposed state estimation method based on the state augmentation strategy has less computational load compared with the traditional one. This paper is an extended version of our conference paper [20]. The main extensions include an additional identification algorithm, a practical simulation example and a real experimental evaluation.

The rest of the paper is organized as follows. Section 2 describes the dual-rate system and problem statements. Section 3 derives a discrete-time state-space model for the dual-rate systems. Section 4 discusses the Kalman filtering algorithm for state estimation, and the SG algorithm or the RLS algorithm for identification of the parameters. Section 5 and 6 provide two illustrative examples and an experimental study on a pilot-scale multiple tank system. Concluding remarks are given in Section 7.

2. System description

Consider a continuous time process P_c in Fig. 1 with the state-space representation:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{B}_c u(t), \\ y(t) = \boldsymbol{C}\boldsymbol{x}(t-\tau) + v(t), \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in R^n$ is the state vector, $u(t) \in R^1$ is the control input which is sampled by a zero-order hold H_{T_1} with sampling period $T_1 = ph$, $y(t) \in R^1$ is the output which is sampled by the sampler S_{T_2} with period $T_2 = qh$, τ is the time delay, $v(t) \in R^1$ is a stochastic noise vector with zero mean, and $\boldsymbol{A}_c$, $\boldsymbol{B}_c$ and $\boldsymbol{C}$ are matrices of appropriate dimensions. Here, h is the positive number called basic period, and p and q are the two positive co-prime integers. Because of the zero-order hold, we have

$$u(t) = u(kT_1) \quad \text{for } kT_1 \le t < (k+1)T_1. \tag{2}$$

For such a dual-rate system, the measurable input–output data is $\{u(kT_1),\ y(kT_2) : k = 0, 1, 2, \ldots\}$. This means $\{u(kT_1 + ih),\ y(kT_2 + jh),$ $i = 1, 2, \ldots, p-1,\ j = 1, 2, \ldots, q-1\}$ are unknown.

The main objective of this paper is to answer the following two questions:

- How to derive the slow-rate state-space model for the dual-rate system with a time delay so that it can be directly identified based on the measured input–output data?
- How to estimate the states and parameters of the system with the time delay based on the Kalman filtering and the SG algorithm or the RLS algorithm?

3. Model derivation

In order to derive the state-space model for the dual-rate system, the continuous process P_c is discretized via the zero-order hold with sampling time h to get P_h as follows:

$$\begin{cases} \boldsymbol{x}(kh+h) = \boldsymbol{A}_h\boldsymbol{x}(kh) + \boldsymbol{B}_h u(kh), \\ y(kh) = \boldsymbol{C}\boldsymbol{x}(kh-dh) + v(kh), \end{cases} \tag{3}$$

where

$$\boldsymbol{A}_h = e^{\boldsymbol{A}_c h}, \quad \boldsymbol{B}_h = \int_0^h e^{\boldsymbol{A}_c t}\, dt\, \boldsymbol{B}_c \quad \text{and} \quad d = \left[\frac{\tau}{h}\right].$$

Theorem 1. *For the system (3), letting $T := pqh$ be the frame period and $m := pq - d$, and assuwming that $pq \geqslant d$, then the state-space model of the dual-rate system can be derived as*

$$\begin{cases} \boldsymbol{x}(kT+T) = \boldsymbol{A}\boldsymbol{x}(kT) + \sum_{j=0}^{q-1}\sum_{i=1}^{p} \boldsymbol{A}_h^{pq-jp-i}\boldsymbol{B}_h u(kT + jT_1), \\ \boldsymbol{x}(kT+T-dh) = \boldsymbol{A}\boldsymbol{x}(kT-dh) + \sum_{j=m+1}^{pq} \boldsymbol{A}_h^{pq+m-j}\boldsymbol{B}_h u(kT+(j-1)h-T) \\ \qquad + \sum_{j=1}^{m} \boldsymbol{A}_h^{m-j}\boldsymbol{B}_h u(kT+(j-1)h), \\ y(kT) = \boldsymbol{C}\boldsymbol{x}(kT-dh) + v(kT), \end{cases} \tag{4}$$

where $\boldsymbol{A} = \boldsymbol{A}_h^{pq}$.

Proof. Substituting k with kpq in (3), we obtain

$$\begin{cases} \boldsymbol{x}(kT+h) = \boldsymbol{A}_h\boldsymbol{x}(kT) + \boldsymbol{B}_h u(kT), \\ y(kT) = \boldsymbol{C}\boldsymbol{x}(kT-dh) + v(kT). \end{cases} \tag{5}$$

This leads to

$$\boldsymbol{x}(kT+T) = \boldsymbol{A}_h^{pq}\boldsymbol{x}(kT) + \sum_{j=0}^{q-1}\sum_{i=1}^{p} \boldsymbol{A}_h^{pq-jp-i}\boldsymbol{B}_h u(kT + jT_1 + (i-1)h). \tag{6}$$

According to (2), we have

$$\boldsymbol{x}(kT+T) = \boldsymbol{A}\boldsymbol{x}(kT) + \sum_{j=0}^{q-1}\sum_{i=1}^{p} \boldsymbol{A}_h^{pq-jp-i}\boldsymbol{B}_h u(kT + jT_1). \tag{7}$$

Combining

$$\boldsymbol{x}(kT+T-dh) = \boldsymbol{A}_h^{pq-d}\boldsymbol{x}(kT) + \sum_{j=1}^{pq-d} \boldsymbol{A}_h^{pq-d-j}\boldsymbol{B}_h u(kT+(j-1)h) \tag{8}$$

and

$$\boldsymbol{x}(kT) = \boldsymbol{A}_h^{d}\boldsymbol{x}(kT-dh) + \sum_{j=pq-d+1}^{pq} \boldsymbol{A}_h^{pq-j}\boldsymbol{B}_h u(kT+(j-1)h-T), \tag{9}$$

yields

$$\boldsymbol{x}(kT+T-dh) = \boldsymbol{A}\boldsymbol{x}(kT-dh) + \sum_{j=m+1}^{pq} \boldsymbol{A}_h^{pq+m-j}\boldsymbol{B}_h u(kT+(j-1)h-T) + \sum_{j=1}^{m} \boldsymbol{A}_h^{m-j}\boldsymbol{B}_h u(kT+(j-1)h). \tag{10}$$

From (10) and (7), the results of (4) can be obtained. □

4. Parameter and state estimation

In this section, the observability canonical model of system (4) is given which is equivalent to the original controllable and observable system and has the least number of the parameters. Then the model ready for the identification of the system is derived. Based on the measurable input–output data, the state estimation can be obtained by using the Kalman filter, and the parameter estimation can be obtained by using the SG algorithm or the RLS algorithm. Due to the time delay, the states of the system cannot be directly estimated by the Kalman filter. A state

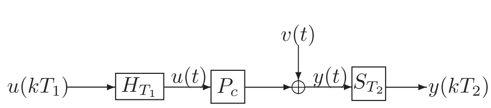

Fig. 1. The general dual-rate sampled-data system.