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Linear control of the flywheel inverted pendulum

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A B S T R A C T

The flywheel inverted pendulum is an underactuated mechanical system with a nonlinear model but admitting a linear approximation around the unstable equilibrium point in the upper position. Although underactuated systems usually require nonlinear controllers, the easy tuning and understanding of linear controllers make them more attractive for designers and final users. In a recent paper, a simple PID controller was proposed by the authors, leading to an internally unstable controlled plant. To achieve global stability, two options are developed here: first by introducing an internal stabilizing controller and second by replacing the PID controller by an observer-based state feedback control. Simulation and experimental results show the effectiveness of the design.

1. Introduction

Underactuated mechanical systems received a lot of interest as they appear in many practical applications such as robotic systems (e.g. mobile robots, flexible-link robots, snake-type robots, and walking robots), aerospace systems (e.g. aircraft, spacecraft, helicopters, rockets and satellites), or marine vehicles (e.g. surface vessels and underwater vehicles). They are characterized by the fact that there are more degrees of freedom than actuators, i.e., one or more degrees of freedom are unactuated [1] presenting challenging control problems to solve operational inconveniences with great interest from the theoretical point of view. Most of the reported works on this kind of mechanical systems approach the problem from a nonlinear perspective [2–5]. The linear approximation around equilibrium points may not, in general, be controllable and the feedback stabilization approach to transform the plant into a linear one, in general, cannot be used. Therefore linear control methods are not used to solve the feedback stabilization problem, not even locally. In the same way, the tracking control problem cannot be transformed into a linear control problem.

But linear control systems are very appealing by their simplicity and easy tuning. The design procedure may have different steps in order to consider different situations but, in any case, a clear understanding of the design parameters is at hand. With this idea in mind, in our previous paper [6], a PID controller was proposed to control a flywheel inverted pendulum (FIP) [7]. Due to the underactuation, a derivative behavior appears at the plant output, so the upper equilibrium position can be reached for any constant or null input value, if the overall system is stable. The problem already reported is that the PID solution is internally unstable.

The paper is organized as follows. First, to fix the problem, the nonlinear model and its approximated linearization around the unstable equilibrium point for this well-known mechanical device are derived. Then, the design of a PID controller to stabilize the plant and to compensate measurement disturbances is reviewed. Looking at the FIP model, it presents an unstable open-loop pole and a zero at the origin. So, even though the input/output behavior of the controlled plant appears to be stable, its internal stability is not achieved. To stabilize the internally unstable controlled plant, two options are considered: first a new control loop is added, keeping the global stability achieved by the initial design and allowing to control the unstable internal variable. The second option is a state feedback control, where cancelation is avoided. This results in a good behavior but it requires full access to the state, thus a state estimator/observer should be implemented. These results are illustrated experimentally by the control of a laboratory prototype. Some comments and future works are outlined in the last section, where the improvements with respect to the previous paper are discussed.

2. Flywheel inverted pendulum

A review on the control of underactuated systems can be found in [1,8] (where the classical inverted pendulum mounted on a cart

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is used as a benchmark of underactuated system), as well as in [9,10], where controllers have been designed by using linearization-based or energy-based methods.

Among the different approaches to control the FIP, a swinging-up control at the unstable equilibrium point, without flywheel angular velocity control, has been reported [11,12], a fuzzy control is reported in [13] and a linear full space state control design using pole assignment has been deeply studied in [14]. As already mentioned, a PID simple solution was proposed in [6].

2.1. The model

A FIP consists of an inverted pendulum pivoting on a frictionless point with a rotating mass on the top. It is an abstraction of a biped robot, with an articulated/motorized joint, where the leg is represented by a bar and the moving body is abstracted as a rotating motor. The reaction torque generated by this rotation allows moving forward/backward the upper part of the pendulum. A local sensor placed at the bottom of the pendulum provides a measurement of its inclination. A picture of such a device [7] is shown in Fig. 1(a) and a schematic diagram of the pendulum is depicted in Fig. 1(b).

A DC motor controlled by the armature voltage is moving the inertia wheel. The main parameters to be considered are the armature resistance (R) and the inductance (L), as well as the torque constant (M).

2.1.1. Lagrangian formulation

Defining by $\{m_p, I_p, \phi\}$ the pendulum mass, its moment of inertia with respect to the base and its angular position with respect to the vertical axis, respectively; and by $\{m_w, I_w, \theta\}$ the flywheel mass, its moment of inertia with respect to its center of mass and its rotation angle, respectively.

The Lagrangian is defined by (1), where E, V denote the kinetic and potential energies, respectively, and q is the generalized coordinates vector of the system, $q=[\phi\ \theta]^T$.

$$L(q,\dot{q}) = E(q,\dot{q}) - V(q,\dot{q}) \tag{1}$$

The system dynamics is derived from the Euler–Lagrange equation (2), where R is the Rayleigh's dissipative function and τ_i are the moments applied to each coordinate (pendulum bar and flywheel)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = \tau_i, \quad i=\phi,\theta \tag{2}$$

The total kinetic energy can be easily expressed as

$$E(q,\dot{q}) = \tfrac{1}{2}[I_p\dot{\phi}^2 + I_w\dot{\theta}^2 + m_w L_p^2\dot{\phi}^2] \tag{3}$$

where I_p and I_w are the inertia moments of the pendulum bar with respect to the fulcrum and the flywheel with respect to its rotation axis, respectively, and L_p is the pendulum length (from the flywheel axis to the fulcrum).

The potential energy is given by

$$V(q,\dot{q}) = m_p g L_c \cos\phi + m_w g L_p \cos\phi \tag{4}$$

where L_c is the pendulum mass center distance to the fulcrum.

Thence, altogether, the Lagrangian is given by

$$\begin{aligned} L(q,\dot{q}) &= \tfrac{1}{2}\alpha_1\dot{\phi}^2 + \tfrac{1}{2}I_w\dot{\theta}^2 - \alpha_2\cos\phi \\ \alpha_1 &= m_w L_p^2 + I_p \\ \alpha_2 &= (m_p L_c + m_w L_p)g \end{aligned} \tag{5}$$

If (2) is applied, taking into account that the generalized moment of the system and the dissipative moments are given by (6), where η_ϕ and η_θ are the friction factors and τ is the external torque applied to the flywheel,

$$\begin{aligned} \tau_\phi &= -I_w\ddot{\theta}, \quad \frac{\partial R}{\partial \dot{q}_\phi} = \eta_\phi\dot{\phi} \\ \tau_\theta &= \tau - I_w\ddot{\phi}, \quad \frac{\partial R}{\partial \dot{q}_\theta} = \eta_\theta\dot{\theta} \end{aligned} \tag{6}$$

then the nonlinear model of the FIP is expressed by the coupled equations:

$$\alpha_1\ddot{\phi} + I_w\ddot{\theta} = \alpha_2\sin\phi - \eta_\phi\dot{\phi} \tag{7}$$

$$I_w(\ddot{\phi} + \ddot{\theta}) = \tau - \eta_\theta\dot{\theta} \tag{8}$$

The input torque $\tau = M i_a$ generated by the electric motor, where i_a is the armature current, is obtained from the differential equation of the armature electric circuit:

$$v_a = R_a i_a + L_a\frac{di_a}{dt} + v_r \tag{9}$$

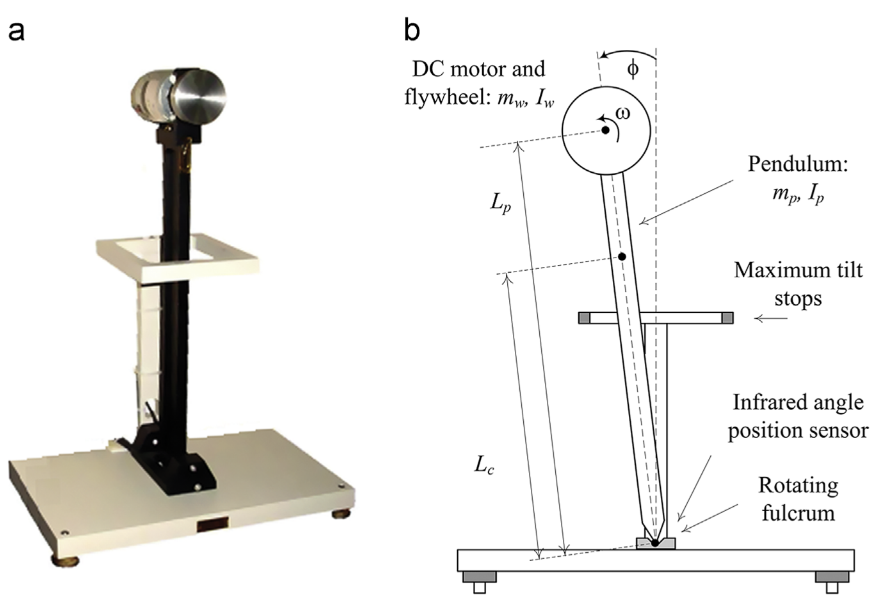

Fig. 1. Flywheel inverted pendulum. (a) Physical device and (b) schematic representation.