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A novel adaptive switching function on fault tolerable sliding mode control for uncertain stochastic systems

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ABSTRACT

A novel switching function based on an optimization strategy for the sliding mode control (SMC) method has been provided for uncertain stochastic systems subject to actuator degradation such that the closed-loop system is globally asymptotically stable with probability one.

In the previous researches the focus on sliding surface has been on proportional or proportional-integral function of states. In this research, from a degree of freedom that depends on designer choice is used to meet certain objectives.

In the design of the switching function, there is a parameter which the designer can regulate for specified objectives. A sliding-mode controller is synthesized to ensure the reachability of the specified switching surface, despite actuator degradation and uncertainties. Finally, the simulation results demonstrate the effectiveness of the proposed method.

1. Introduction

This paper will investigate the design of reliable SMC for uncertain stochastic systems with possible occurrence of actuator faults.

Sliding mode control is an effective robust scheme for incomplete modeled or uncertain systems, whose key feature relies on its complete insensitiveness to both parameter variations and external disturbances. Hence, in the past decades, a lot of important developments on SMC have been obtained [1–13].

As is well-known, the actuator degradation in actual physical systems is usually inevitable, and often yields performance degradation or even instability. Therefore, how to maintain an acceptable stability/performance for the closed-loop systems against actuator or sensor failures has been a long-standing and active research topic. In the literature, the design of reliable control systems can be broadly classified as the active approach and the passive approach. In the first type, faults are detected and identified by a fault detection and diagnosis (FDD) mechanism, and controllers are re-configured, such as those in [14–19]. In contrast, the passive methodology designs a reliable controller with a fixed structure by means of actuator redundancy. By taking possible actuator faults into account during the design of the controller, passive reliable systems can attain the stability and performance requirement even in the presence of actuator faults. Moreover, various passive reliable techniques have also been developed, for example, linear-quadratic state-feedback control [20], pre-compensator [21], H_1 disturbance attenuation [22], control allocation [23], and adaptive control [24–26].

It is known that a stochastic system has extensive applications in practice, and the SMC of stochastic systems receives much attention [27–29]. However, little work has been carried out on the reliable problem of SMC for uncertain stochastic systems subject to actuator faults.

A common feature in the last corresponding works is related to choosing sliding surface which is mostly chosen as a proportional or proportional-integral function of states [30–32].

In the mentioned sliding surfaces we cannot discuss about the effect of changing a parameter of switching function on performance of the system, strongly. Furthermore these sliding manifolds have been proposed without expressing a reason and defining a specific goal. In this work, from a degree of freedom that depends on designer choice is used to meet certain objectives which were neglected in other recent researches. In other words, sliding surface is extracted based on a systematic strategy. In this method the problem of choosing sliding surface is converted to a virtual optimal control problem such that the constructed sliding surface minimizes a quadratic performance index. In this methodology there is a possibility of regulating the distance of state trajectories from sliding surface and the amount of energy consumption that has wealth importance in control strategies.

Eventually, an adaptive sliding mode controller is synthesized, which can ensure the reachability of the aforementioned sliding surface. Additionally, sufficient conditions are derived via the stochastic Lyapunov method and linear matrix inequalities such

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that the closed-loop system is ensured to be asymptotically stable in probability one, not only when control component is in good working condition but also in the presence of actuator faults.

Notations $|.|$, $\|.\|$ denote, respectively absolute value and the Euclidean norm of a vector or its induced matrix norm. For a real matrix, $M>0$ means that M is symmetric and positive definite, and I is used to represent an identity matrix of appropriate dimensions. $\lambda_i(.)$ shows the i-th Eigen value of a matrix. (Ω, F, P) is a probability space with Ω the sample space, and F the σ-algebra of subsets of the sample space, and P is the probability measure. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. System description and preliminaries

In this work, consider the stochastic nonlinear single-input systems which are established in the probability space (Ω, F, P) and described by the Itô stochastic differential equation as follows [32]:

$$dx=[(A+\Delta A(t))x+B(u_F(t)+f(t))]dt+Dg(x,t)de(t) \quad (1)$$

where $x\in\mathbb{R}^n$, $u_F\in\mathbb{R}$ and $e(t)$ are, respectively, the states of system, the faulty control input, and l-dimensional Brownian motion. Here, $A\in\mathbb{R}^{n*n}$, $B\in\mathbb{R}^{n*1}$, $D\in\mathbb{R}^{n*m}$ are known real constant matrices of appropriate dimensions. Without loss of generality, it is assumed that the pair (A,B) is controllable. Moreover, matrix ΔA is for parameter uncertainty, $f(t)$ is an unknown time varying function and $|f(t)|<r$ where $r>0$ is a known scalar and the unknown function $g(x,t)\in\mathbb{R}^{ml}$ denotes nonlinear uncertainty, and it is assumed that the admissible uncertainties satisfy

$$\Delta A=EF(t)H$$

$$\text{trace}[g^T(x,t)g(x,t)]\le N x(t)^2 \quad (2)$$

where E, H and N are known as real constant matrices, and $F(t)$ is an unknown matrix function satisfying $F^T(t)F(t)\le I$.

In system (1) it is assumed that partial actuator degradation may occur, and this can be modeled as follows:

$$u_F(t)=(1-\mu)u(t) \quad (3)$$

where μ satisfies $0\le\mu_{min}\le\mu\le\mu_{max}<1$. The scalar μ usually termed as the effectiveness loss value of the actuator, which denotes the decrease in the effectiveness of the specified actuator.

Remark 1. It can be seen that if $\mu=1$ the control gain $B(1-\mu)$ will not satisfy the condition of full column rank that is a general assumption for the design of SMC. Hence, it is reasonable for SMC design to only consider the partial actuator degradation.

Definition 1. The equilibrium solution, $x_t=0$ of the stochastic differential Eq. (1) with $u(t)=0$ is said to be globally asymptotically stable (with probability one) if for any $\tau\ge 0$ and $\varepsilon>0$,

$$\lim_{x\to 0} P\left\{\sup_{\tau<t}|x_t^{\tau,x}|>\varepsilon\right\}=0,\quad P\left\{\lim_{t\to\infty}|x_t^{\tau,x}|=0\right\}=1$$

where $x_t^{\tau,x}$ denotes the solution at time t of a stochastic differential equation starting from the state x at time τ for $\tau<t$.

Lemma 1. *The trivial solution of the stochastic differential equation*

$$dx(t)=a(t,x)dt+b(t,x)dw(t)$$

with $a(t,x)$ and $b(t,x)$ sufficiently differentiable maps, is globally asymptotically stable (with probability one) if there exists a positive definite radially unbounded function $V(t,x)$, and satisfies

$$\mathscr{L}V=\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x}a(t,x)+\frac{1}{2}\text{trace}\left\{b(t,x)^T\frac{\partial^2 V}{\partial x^2}b(t,x)\right\}<0 \quad \text{for } x\ne 0 \quad (4)$$

3. Sliding surface construction

In this work, sliding manifold is chosen as

$$s=\frac{1}{2}\,\text{Ln}\,\frac{\rho(t)-R^{-1}B^TPx(t)}{\rho(t)+R^{-1}B^TPx(t)}=0 \quad (5)$$

where $\rho(t)$ is a positive scalar to be regulated later so that $\rho(t)>|R^{-1}B^TPx|$ which leads to $s(t)$ that is real and feasible, $R>0$ is a scalar that is determined for some objectives and $P>0$ to be designed later is a symmetric matrix that meets

$$(A+BZ)^TP+P(A+BZ)-PBR^{-1}B^TP+Q=0$$

where $Q>0$ is a symmetric matrix and Z is a row vector that is chosen so that $(A+BZ)$ is Hurwitz and $\min_i(|\lambda_i(A+BZ)|)\gg R^{-1}B^TPB(1-\mu_{max})$.

This sliding function is extracted from an optimization procedure as explained in the following:

If $u(t)=Zx(t)+\rho(t)\tanh(s)$ is applied to the nominal system $dx(t)=(Ax(t)+Bu(t))dt$ we have

$$dx(t)=[Ax+B(Zx+\rho(t)\tanh(s))]dt=[(A+BZ)x+B\rho(t)\tanh(s)]dt \quad (6)$$

If we define

$$A+BZ=\tilde{A} \quad (7)$$

$$\rho(t)\tanh(s)=v(t)$$

Then $dx(t)=(Ax(t)+Bu(t))dt$ can be rewritten as

$$dx(t)=(\tilde{A}x(t)+Bv(t))dt \quad (8)$$

For dynamical system (8), by choosing performance index

$$J=\int_0^\infty \{x^T(t)Qx(t)+Rv^2(t)\}\,dt \quad (9)$$

where $Q>0$ is a symmetric matrix and $R>0$ is a scalar.

The virtual optimal controller is extracted as

$$v(t)=-R^{-1}B^TPx(t) \quad (10)$$

where P satisfies $\tilde{A}^TP+P\tilde{A}+PBR^{-1}B^TP-Q=0$

Comparing (7) and (10)

$$\rho(t)\tanh(s(t))=-R^{-1}B^TPx(t) \quad (11)$$

which yields

$$s(t)=\frac{1}{2}\,\text{Ln}\,\frac{\rho(t)-R^{-1}B^TPx(t)}{\rho(t)+R^{-1}B^TPx(t)} \quad (12)$$

This switching function has multiple benefits. By increasing R one may reduce control effort or distance of state trajectories from sliding surface.

It follows from (1) and (12) that

$$s=\frac{1}{2}\,\text{Ln}\,\frac{\rho(t)-R^{-1}B^TPx(t_0)-\int_0^t R^{-1}B^TP\{(A+\Delta A(\tau))x+B(u_F+f(\tau))\}d\tau-\int_0^t R^{-1}B^TPDg(x,\tau)de(\tau)}{\rho(t)+R^{-1}B^TPx(t_0)+\int_0^t R^{-1}B^TP\{(A+\Delta A(\tau))x+B(u_F+f(\tau))\}d\tau+\int_0^t R^{-1}B^TPDg(x,\tau)de(\tau)} \quad (13)$$

where $x(t_0)$ is the initial state at $t_0\ge 0$ and the last term in numerator and denominator of logarithmic function is an Itô stochastic integral.

Under the condition that $B^TPD=0$, the sliding function (13) reduces to

$$s=\frac{1}{2}\,\text{Ln}\,\frac{\rho(t)-R^{-1}B^TPx(t_0)-\int_0^t R^{-1}B^TP\{(A+\Delta A(\tau))x+B(u^F+f(\tau))\}d\tau}{\rho(t)+R^{-1}B^TPx(t_0)+\int_0^t R^{-1}B^TP\{(A+\Delta A(\tau))x+B(u^F+f(\tau))\}d\tau} \quad (14)$$

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