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A discrete time-varying internal model-based approach for high precision tracking of a multi-axis servo gantry

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ABSTRACT

In this paper, we consider the discrete time-varying internal model-based control design for high precision tracking of complicated reference trajectories generated by time-varying systems. Based on a novel parallel time-varying internal model structure, asymptotic tracking conditions for the design of internal model units are developed, and a low order robust time-varying stabilizer is further synthesized. In a discrete time setting, the high precision tracking control architecture is deployed on a Voice Coil Motor (VCM) actuated servo gantry system, where numerical simulations and real time experimental results are provided, achieving the tracking errors around 3.5‰ for frequency-varying signals.

1. Introduction

One of the central topics in the control of mechatronics is trajectory tracking, which has important applications to a large class of high precision manipulations, such as machine tools [1], lithography machining in semiconductors [2], and track seeking of HDDs [3]. Significant efforts have been devoted to various aspects -of tracking control theory in the past several decades (see, for example [4-11]). As one of the most investigated approaches, the internal model-based control method has emerged as a fundamental technique for tracking and/or rejecting periodic signals generated by autonomous systems.

Although the internal model-based control theory for LTI systems has been well established [4], the results for LTV (Linear Time-Varying) systems are not available, due to the fundamental challenges of constructing a time-varying internal model to render the error-zeroing subspace invariant, and a robust time-varying stabilizer for the augmented time-varying system. We refer to [7-9] for some recent advances of internal model-based design for LTV systems.

It is worth mentioning that a systematic design method for the construction of time-varying internal model has been proposed in [10,11] in both input/output and state-space representations. The implementations of the above algorithms, however, still face the challenge of designing robust and low-order stabilizers. Notice that some attempts have been made via LPV (Linear Parameter-Varying) design approaches in continuous time settings, for example [12–14]. In order to make the control architecture more implementable for tracking sophisticated signals in real applications, it is desirable to cast the internal model-based control framework in a discrete time setting, which would greatly reduce the computational burdens and avoid numerical issues. Very recently a discrete time tracking controller designs have been proposed in [15,16], which are non-trivial extension of the results in continuous time settings [13,14].

In the present paper, we investigate the discrete time-varying internal model-based design by resorting to the recently developed parallel structure for time-varying internal model control [14], which can be considered as the counter part of the continuous time domain results in [14]. The tracking control algorithm is deployed for a high precision X–Y servo gantry driven by VCMs (Voice Coil Motors), which represents many important industrial applications such as laser beam steering, PCB laser marking, and advanced imaging systems [17].

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The rest of the paper is organized as follows: the problem formulation of the tracking problem under consideration and some preliminaries on a time-varying internal model design are briefly discussed in Section 2. In Section 3, a discrete time-varying robust stabilizer design is discussed based on the parallel connection with the internal model unit. The simulation and experimental results for controlling a servo gantry platform are given in Section 4 to demonstrate the proposed control algorithm in the discrete time-varying setting, followed by conclusions in Section 5.

2. Problem formulation and preliminaries

2.1. Problem formulation

We in this work consider discrete LTI plant models of the form

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ y(k) &= Cx(k) \\ e(k) &= y(k)+r(k), \end{aligned} \tag{1}$$

with plant state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}$, output $y \in \mathbb{R}$, reference $r \in \mathbb{R}$, and regulated error $e \in \mathbb{R}$ satisfying the following assumption.

Assumption 2.1. The triplet (A, B, C) is controllable and observable.

The reference $r(k)$ to be tracked is generated by an LTV exosystem of the form

$$\begin{aligned} w(k+1) &= S(k)w(k) \\ r(k) &= Q(k)w(k) \end{aligned} \tag{2}$$

with exogenous state $w \in \mathbb{R}^\rho$. The exosystem under consideration is characterized by the following assumption.

Assumption 2.2. The trajectories $w(k)$ in the forward and backward directions of time are stable in the sense of Lyapunov, and the pair $(Q(\cdot), S(\cdot))$ is uniformly observable.

Note that the problem of *asymptotically tracking* complicated signals generated by the time-varying exosystem (2) has yet to be solved due to its major difficulty of time-varying internal model-based design. Towards a complete and implementation orientated solution, a novel controller architecture is proposed in Fig. 1, which consists of a time-varying internal model unit and a time-varying robust stabilizer.

2.2. Preliminaries of a time-varying internal model construction

Recently, it is shown in [10,11] that for time-varying systems the design of the time-varying internal model can be constructed by a two-step way, i.e., (1) immersing the exogenous signal r in the place of u_r (see Fig. 1); (2) making the I/O mappings between the subsystem $(\Phi_1(\cdot), \Psi_1(\cdot), \Gamma_1(\cdot))$ and the plant model the same.

Along this thread, the design of a simple and low order stabilizer for the resulting time-varying systems remains a great challenge. Aiming at resolving this difficulty, we consider a novel compensator structure where the internal model unit and the stabilizer are interconnected in parallel (see Fig. 1, and the advantages of adopting such a paralleled structure will be clear in the later sections). From Fig. 1, it is readily seen that the internal model unit admits the following form:

Internal model subsystem 1:

$$\begin{aligned} \xi_1(k+1) &= \Phi_1(k)\xi_1(k)+\Psi_1(k)u(k) \\ u_r(k) &= \Gamma_1(k)\xi_1(k) \end{aligned} \tag{3}$$

and *Internal model subsystem* 2:

$$\begin{aligned} \xi_2(k+1) &= \Phi_2(k)\xi_2(k)+\Psi_2(k)(-u_r(k)) \\ u_{im}(k) &= \Gamma_2(k)\xi_2(k)+D_2(k)(-u_r(k)) \end{aligned} \tag{4}$$

with the internal model state $(\xi_1, \xi_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$, embedded input $u_r \in \mathbb{R}$, and internal model input $u_{im} \in \mathbb{R}$.

Note that the detailed design of system (3)–(4) can be referred to [11], and the main results are listed as follows:

(1) By solving the following *algebraic* Sylvester equation:

$$(\mathcal{O}_{\Phi_1(k)} \ \ \mathcal{C}_{\Psi_1(k)}) \begin{pmatrix} 1 \\ q(k) \\ p(k) \end{pmatrix} = \mathcal{O}_{S(k)} \begin{pmatrix} 1 \\ q(k) \end{pmatrix},$$

the signal r is embedded in the place of u_r; where $q(k)$ and $p(k)$ collect time-varying coefficients of $\Phi_2(\cdot)$ and $\Gamma_2(\cdot)$ in controller canonical form respectively, and $\mathcal{O}_{S(k)}$ and $\mathcal{C}_{\Psi_1(k)}$ are defined in Appendix equation (24), and $\mathcal{O}_{\Phi_1(k)}$ is defined similar to that of $\mathcal{O}_{S(k)}$. More detailed explanations are referred to [11] and an illustrative example is given in Section 4.2 for the reader's convenience.

(2) By choosing the nominal values of the plant model as the subsystem $(\Phi_1(\cdot), \Psi_1(\cdot), \Gamma_1(\cdot))$, the exosystem with the required error zeroing input u_{ff}, i.e.,

$$\begin{aligned} w(k+1) &= S(k)w(k) \\ u_{ff}(k) &= R(k)w(k) \end{aligned} \tag{5}$$

is immersed [18,10,11] into

$$\begin{aligned} \begin{pmatrix} \xi_1(k+1) \\ \xi_2(k+1) \end{pmatrix} &= \Phi(k) \begin{pmatrix} \xi_1(k) \\ \xi_2(k) \end{pmatrix} \\ u_{im}(k) &= \Gamma(k) \begin{pmatrix} \xi_1(k) \\ \xi_2(k) \end{pmatrix}, \end{aligned} \tag{6}$$

where

$$\Phi(k) = \begin{pmatrix} \Phi_1(k)-\Psi_1(k)D_2(k)\Gamma_1(k) & \Psi_1(k)\Gamma_2(k) \\ -\Psi_2(k)\Gamma_1(k) & \Phi_2(k) \end{pmatrix},$$

$$\Gamma(k) = (-D_2(k)\Gamma_1(k) \ \ \Gamma_2(k)),$$

and $u_{im}(k)$ is the desired input to keep error $e(k)=0$ (see [11]

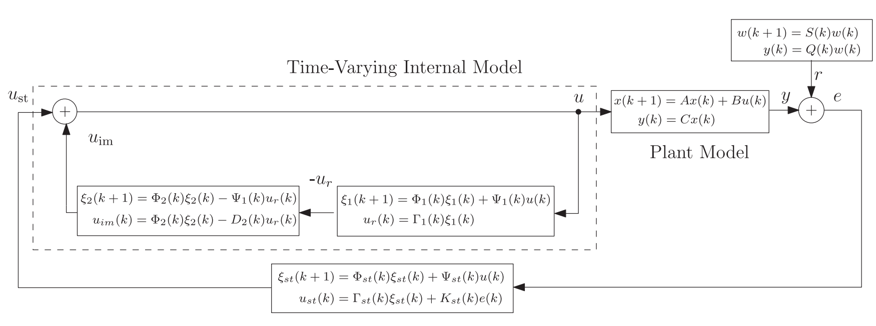

Fig. 1. The block diagram of a parallel connected time-varying internal model-based controller.