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Research Article

Analytical design of fractional-order proportional-integral controllers for time-delay processes

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ABSTRACT

A new design method of fractional-order proportional-integral controllers is proposed based on fractional calculus and Bode's ideal transfer function for a first-order-plus-dead-time process model. It can be extended to be applied to various dynamic models. Tuning rules were analytically derived to cope with both set-point tracking and disturbance rejection problems. Simulations of a broad range of processes are reported, with each simulated controller being tuned to have a similar degree of robustness in terms of resonant peak to other reported controllers. The proposed controller consistently showed improved performance over other similar controllers and established integer PI controllers.

1. Introduction

Fractional-order dynamic systems are useful in presenting various stable physical phenomena with anomalous decay [1]. Fractional calculus (i.e. fractional integro-differential operators) is a generalization of integration and differentiation to non-integer orders. It is obtained from ordinary calculus by extending ordinary differential equations (ODE) to fractional-order differential equations (FODE). Similarly, a fractional-order proportional-integral-derivative (FOPID) controller is a generalization of a standard (integer) PID controller; its output is a linear combination of the input and the fractional integral or derivative of the error [2]. It affords more flexibility in PID controller design due to its five controller parameters (instead of the standard three): proportional gain, integral gain, derivative gain, integral order, and derivative order. However, the tuning rules of fractional-order PID (FOPID) controllers are much more complex than those of standard (integer) PID controllers with only three parameters. Several design methodologies of FOPID controllers have been introduced to facilitate their use: Bode first reported fractional structures in feedback-loops [3,4]. This was extended by Barbosa et al. [5], who reported a feedback amplifier obtained by considering a feedback-loop in terms of the performance of a closed-loop that was invariant to changes of amplifier gain. However, this concept was not rigorously developed and remained neglected for decades. Oustaloup [6] introduced fractional-order algorithms for the control of dynamic systems based on non-integer derivatives and demonstrated significant improvement of CRONE (Commande Robuste d'Ordre Non Entier) controllers over integer PID controllers. CRONE controllers are obtained using a rational form and the major differences between the three generations lie in the design of the open-loop, the slope of which depends on consideration of plant uncertainty. The generalized PID controller, $PI^{\lambda}D^{\mu}$, that involves a fractional-order integrator (λ) and a fractional-order differentiator"body (μ), was suggested by Podlubny [7]. The two extra parameters (λ and μ) give this type of controller improved flexibility over integer PID controllers, giving it much industrial applicability [8,9]. Tuning methods of $PI^{\lambda}D^{\mu}$ controllers can be generally classified as either analytic or heuristic [10,11].

Most analytical methods are tuned by considering the nonlinear objective function, which is depended on user-imposed specifications [5,11–14]. Barbosa et al. [5] introduced the tuning of integer PID controller by considering the system similar to a fractional-order system that is done by using the ISE error minimization. Monje et al. [13] proposed five conditions taking into account phase and gain margins specifications, as well as the constraints over the sensitivity functions. Valério and Da costa [11] considered Ziegler-Nichols-type tuning rules for the first order plus time delay (FOPDT) processes. In addition, F-MIGO (i.e., peak sensitivity constrained integral gain optimization for the fractional-order PI control system) method [14] was developed for the FOPDT class of dynamic systems, which is generalized to

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handle the FOPI control system from the so-called MIGO (i.e. Ms, the maximum sensitivity constrained integral gain optimization) developed by Åström et al. [15–17]. In accordance with the F-MIGO method, the Nyquist plot of the open-loop transfer function lies outside the circle such that it encloses both the Ms and Mp circles. This circle has the center and radius as shown in [14]. Furthermore, Vinagre [18] suggested setting $\lambda=\mu$ and imposing a phase margin at gain crossover frequency. Caponetto et al. [19] proposed a method selecting freely a $\lambda=\mu>1$, which is allowed freely choosing controller parameters by imposing a phase margin at gain crossover frequency. A fractional PI controller was tuned by the combination of gain and phase margin requirements with a flat phase for the open-loop at critical frequency introduced by Chen et al. [20]. The internal model control (IMC) methodology can be also used in some cases to obtain PID or fractional PID controllers [11].

This work proposes a new analytic method of FOPI controller design for enhanced set-point tracking and disturbance rejection responses of processes with time delays. It is based largely on fractional calculus and Bode's ideal transfer function. By using frequency domain, the proposed FOPI tuning rules can be directly derived for first-order-plus-dead-time (FOPDT) models and can be applied to various process models.

The paper is organized as follows. The fundamentals of fractional calculus and their application for obtaining the FOPI controller are given in Section 2. In Section 3, the generalized FOPI controller tuning rules and some important robustness and performance indices are introduced. Section 4 gives some illustrated examples, where a comparison with other design methods is presented. Some important guidelines are introduced in Section 5. Conclusions are given in Section 6.

2. Preliminaries

This section introduces some fundamentals of fractional calculus, the problem statement required to understand fractional systems, and the examined controller.

2.1. Fractional calculus

Fractional calculus [21] is a generalization of ordinary calculus. It develops a functional operator, D, associated to the order of an operation ν ($\nu\in\Re$) not restricted to integers that generalizes usual derivatives (for positive ν) and integrals (for negative ν). There are various definitions of fractional differentiation. However, the most commonly used is the Riemann–Liouville definition [9,21], which is generalized by two equalities easily proven for integer orders:

$$ {}_aD_t^{\nu}f(t)=\frac{1}{\Gamma(n-\nu)}\frac{d^n}{dt^n}\int_a^t\frac{f(\tau)}{(t-\tau)^{\nu-n+1}}dt, n-1<\nu<n \tag{1}$$

where $\Gamma(\bullet)$ denotes Euler's gamma function. a and t are the limits.

Note that the Laplace transform of the fractional derivative/integral in (1) follows the rule for zero initial condition for order ν $(0<\nu<1)$:

$$L\{{}_aD_t^{\pm\nu}f(t)\}=s^{\pm\nu}F(s) \tag{2}$$

The initial conditions imply that a dynamic system described by differential equations involving fractional derivatives gives rise to transfer functions with fractional powers of s. This is described further elsewhere [21].

2.2. Integer order approximation

For fractional-order controllers to be used for simulation and hardware applications with transfer functions that involve fractional orders of s, the transfer function should be approximated as an integer-order transfer function with similar behavior, which includes an infinite number of poles and zeros. Nevertheless, reasonable approximations can be obtained with finite numbers of poles and zeros. In this case, the Oustaloup continuous integer-order approximation [6] based on the recursive distribution of poles and zeros is employed here:

$$s^{\nu}\cong k\prod_{n=1}^{N}\frac{1+(s/\omega_{z,n})}{1+(s/\omega_{p,n})}, \nu>0 \tag{3}$$

Eq. (3) is valid over the frequency range $[\omega_l,\omega_h]$, where the gain, k, should be adjusted for both sides of (2) to have unity gain at the gain crossover frequency, s^{ν} (i.e. $\omega_c=1$ rad/s). Eight poles and zeros (i.e. $N=8$) is chosen, since ω_l and ω_h are respectively $0.001\omega_c$ and $1000\omega_c$. It is important to note that low values result in simpler approximations, but may cause ripples in both gain and phase behaviors. The ripples can be functionally neglected by increasing N, and hence also increasing computation costs. In addition, frequencies of zeroes and poles in (3) are given as follows:

$$\omega_{z,l}=\omega_l\sqrt{\eta} \tag{4a}$$

$$\omega_{p,n}=\omega_{z,n}\alpha,\quad n=1,2,\ldots N \tag{4b}$$

$$\omega_{p,n+1}=\omega_{p,n}\eta,\quad n=1,2,\ldots N-1 \tag{4c}$$

$$\alpha=(\omega_h/\omega_l)^{\nu/N} \tag{4d}$$

$$\eta=(\omega_h/\omega_l)^{(1-\nu)/N} \tag{4e}$$

It can be dealt with inverting (3) when $\nu<0$. But in the case $|\nu|>1$, the approximation will be unsatisfactory. Therefore, it is common to split fractional power of s as follows:

$$s^{\nu}=s^n s^{\delta},\quad \nu=n+\delta\wedge n\in Z\wedge\delta\in[0;1] \tag{5}$$

2.3. FOPI controller

Fractional calculus gives the fractional integro-differential equation of a FOPI controller as:

$$u(t)=\mathrm{K_C}e(t)+\mathrm{K_I}D_t^{-\lambda}e(t),\quad(\lambda>0) \tag{6}$$

where K_C and K_I represent the proportional and integral terms of the FOPI controller, respectively. λ is the fractional order of the integral.

The continuous transfer function of the FOPI controller can be obtained by Laplace transformation:

$$G_C(s)=K_C+\frac{K_I}{s^{\lambda}} \tag{7}$$

The FOPI controller has three parameters (K_C,K_I, and λ) to tune, since the fractional order λ is not necessarily integer. An integer PI controller is a special case of this FOPI controller where $\lambda=1$. This expansion provides more flexibility in achieving control objectives. However, it is often complicated by requiring a non-linear objective function and user-defined constraints to obtain controller parameters that satisfy some specified performance criterion.

By substituting $s=j\omega$ into (7), the FOPI controller is represented in the frequency domain as:

$$G_C(j\omega)=K_C+\frac{K_I}{(j\omega)^{\lambda}} \tag{8}$$

The fractional power of $j\omega$ can be written as

$$(j\omega)^{\lambda}=\omega^{\lambda}j+=\omega^{\lambda}[e^{j[(\pi/2)+2n\pi]}]^{\lambda}=\omega^{\lambda}[e^{j[(\pi/2)\lambda+2n\lambda\pi]}] \tag{9}$$

where $n=0, \pm(1/\lambda), \pm(2/\lambda), \ldots, \pm(m/\lambda)$. Therefore, the following convenient form is obtained:

$$(j\omega)^{\lambda}=\omega^{\lambda}(\cos\gamma_I+j\sin\gamma_I),\quad \gamma_I=\frac{\pi\lambda}{2} \tag{10}$$