ISA Transactions 52 (2013) 629–637

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Robust state estimation using desensitized Divided Difference Filter

Christopher D. Karlgaard *, Haijun Shen

Analytical Mechanics Associates, Inc., 303 Butler Farm Road, Suite 104A, Hampton, VA 23666, United States

ARTICLE INFO

Article history:
Received 23 October 2012
Received in revised form
9 April 2013
Accepted 27 April 2013
Available online 28 May 2013
This paper was recommended for publication by Dr. Q.-G. Wang.

Keywords:
Robust filtering
Optimal filtering
Divided difference filtering
Parameter uncertainty

ABSTRACT

This paper develops a robust divided difference filtering approach based on the concept of Desensitized Kalman Filtering. The filters are formulated using a minimum variance cost function, augmented with a penalty function consisting of a weighted norm of the state sensitivities. Solutions are provided for first and second-order Divided Difference Filters. The resulting filters are non-minimum variance but exhibit reduced sensitivity to deviations in the assumed plant model parameters. The proposed algorithms are demonstrated using Monte Carlo simulation techniques for an induction motor state estimation problem with parameter uncertainties.

1. Introduction

The Kalman Filter and its variations such as the Extended Kalman Filter (EKF) are well-known state estimation techniques in wide use in a variety of applications such as vehicle navigation, target tracking, automotive vehicle state estimation, chemical processing, atmospheric data assimilation, and many other areas. Over the last decade, the class of Sigma Point Kalman Filters (SPKF) [1–4] have emerged as replacements for the industry standard EKF for nonlinear estimation problems. These Sigma Point Kalman Filters include the Central Difference Filter (CDF) [1], the Divided Difference Filter (DDF) [2], and the Unscented Kalman Filter (UKF) [3,4]. Like the basic Kalman Filter, the SPKFs seek to determine a state estimate that minimizes the posterior covariance. The SPKF technique differs from the standard Kalman Filter in the sense that the SPKFs do not linearize the dynamic system for the propagation, but instead propagate a cluster of points centered around the current estimate to form improved approximations of the conditional mean and covariance. Specifically, the CDF makes use of first-order finite-difference approximations of the plant and measurement models. The DDF makes use of multi-dimensional interpolation formulas, rather than the Taylor series expansions in use in Kalman Filter algorithms. The UKF uses the Unscented Transformation, which attempts to approximate the prior and posterior distributions, rather than the nonlinear plant model, using a deterministic sampling approach. As a result of these approaches, the class of SPKFs do not require knowledge or existence of the partial derivatives of the system dynamics and measurement equations. SPKFs have the additional advantage over the basic Kalman Filter in that they can easily be extended to determine second-order solutions to the minimum variance filtering problem, which increases the estimation accuracy when the system and/or measurement equations are nonlinear.

The filter performance can be extremely sensitive to deviations in the assumed plant and measurement models. Such deviations can be in the form of model parameter uncertainty or uncertainties in the assumed process and measurement noise statistics, such as non-Gaussian errors. Past research has been conducted to develop *robust* filtering approaches that are less sensitive to deviations in the assumptions inherent to the Kalman Filter. One of the first approaches proposed for dealing with parameter uncertainty was introduced by Schmidt [5]. Schmidt proposes two techniques, the first being a state augmentation approach in which the uncertain parameters are estimated along with the states in the filter. In the second approach, the parameters are considered as structured process and/or measurement noise, and the state estimate error covariance matrix is adjusted in order to account for the resulting uncertainties. The latter method has become known as the Kalman-Schmidt Filter [6] or the "consider" Kalman Filter [7]. This type of filter has advantages over the state augmentation approach in that reduced-order filters can be utilized, and potential observability problems can be mitigated by accounting for the uncertainty in the parameters rather than attempting to estimate them directly. Another drawback of the

* Corresponding author. Tel.: +1 757 865 0000.
E-mail addresses: karlgaard@ama-inc.com (C.D. Karlgaard), shen@ama-inc.com (H. Shen).

0019-0578/$ - see front matter © 2013 ISA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.isatra.2013.04.009

state augmentation approach is that it can introduce nonlinearity when applied to linear systems, which implies that the resulting filters are suboptimal. A notionally similar approach to the Kalman-Schmidt filter is taken in Ref. [8], where reduced-order robust linear filters are derived by treating the parameter uncertainties as state and measurement dependent random errors. Recently, Kai et al. [9] have developed a robust Kalman Filter solution for nonlinear discrete-time systems with stochastic multiplicative uncertainty in the state dynamics and measurement models.

Much attention has been given to the development of robust filters with norm-bounded parameter uncertainty [10–15]. Specifically, Xie et al. [10] develop a robust steady-state Kalman Filter design for linear discrete-time systems with norm-bounded parameter uncertainty in the state and output matrices. A similar approach to the robust H_∞ filtering problem with norm-bounded parameter uncertainty are investigated in Ref. [11]. Dong and You [12] develop a robust Kalman Filter for linear discrete-time systems with norm-bounded parameter uncertainty in the state and output matrices, with unknown but bounded uncertainty in the process and measurement noise statistics. Souto and Ishihara [13] extend the work of Dong and You to the case of unknown correlations between the process and measurement noise. Recently, Ref. [14] develops a robust Kalman Filter for norm-bounded parameter uncertainties using a modified Riccati equation approach. Mahmoud et al. [15] investigates robust filtering approaches for linear systems with Markovian jump parameters, with norm-bounded parameter uncertainties in the state and measurement equations. A State Dependent Riccati Equation approach has also recently been applied to the H_∞ filtering problem with norm-bounded parameter uncertainty in Ref. [16].

Other developments of robust filtering approaches include the Smooth Variable Structure Filter (SVSF) derived based on the concepts of variable structure control [17]. The SVSF assumes a predictor-corrector structure, and tries to guide the state estimates along a so-called existence subspace whose width is bounded by the process and measurement noise levels. An approach based on Multiple Model Adaptive Estimation (MMAE) is proposed on Ref. [18]. In this approach, a bank of Kalman Filters with different assumptions on the parameter values are implemented in parallel. A weighted average of the filter outputs forms the basis for the state estimate. Ref. [19] proposes a model parameter estimation approach based on the principles of fuzzy control, and a neural network model identification approach is developed in Ref. [20]. Robust filtering for problems with faulty measurement data is investigated in Refs. [21,22]. Estimators for classes of problems with time delays are developed in Refs. [23,24].

This paper proposes a robust Divided Difference Filter for systems with vector parameter uncertainty, without known bounds or known statistical information. The solution is obtained using a robust optimal control technique known as *Desensitized Optimal Control* (DOC). Historically, optimal control problems are formulated such that the user specifies a single scalar cost index. Then, optimization techniques are applied to generate control logic and the associated optimal solution that yields the best performance for exactly the cost function and model configuration that were assumed in the calculation. In practice, however, the real-life data often differs from the assumed model, and it is desirable if certain user-specified characteristics of the optimal solution, such as performance in terms of the original cost index or the width of certain safety margins associated with the obtained control logic, did not deteriorate too rapidly as the model is changed, or as external perturbations are added. In practice one would prefer a control law that sacrifices a reasonable amount of performance for the nominal case, in favor of reducing the rate at which user-defined safety and performance measures deteriorate when some system parameters are changed or perturbations are experienced. The DOC methodology is a set of theory that enables the design of such trade-off solutions in an optimal fashion.

The DOC methodology was originally introduced in 1996 by Seywald and Kumar in Ref. [25]. The basic idea is to embed the reference solution in a field of neighboring paths onto which the solution is allowed to "jump" (governed by a linear feedback control law) in case perturbations are encountered along the nominal trajectory. When a physical quantity is identified whose sensitivity with respect to perturbations is to be reduced, the associated sensitivity can be derived mathematically. The objective of reducing the sensitivity can be represented by a performance index J_s. Then, J_s and the original performance index J_0 are minimized concurrently, resulting in a multi-objective optimization problem. A penalty factor c_0 is introduced to consolidate the two competing objectives into one performance index, given as

$$J = J_0 + c_0 J_s \tag{1}$$

Minimizing J results in a different nominal trajectory, which is an optimal compromise between sacrificing the performance and gaining sensitivity reduction.

This methodology has been extended based on the original formulation, and been successfully applied to a wide range of trajectory optimization problems. Ref. [26] addresses the optimal control problem with control constraints and a vertical rocket landing problem, where trajectories are obtained that have reduced sensitivity to the perturbation on the rocket thrust level. Ref. [27] presents optimal orbital insertion trajectories such that the sensitivity of the achieved target orbit with respect to the perturbations on the air density is reduced. Ref. [28] desensitized the Mars Entry trajectory with respect to initial orbit insertion errors and the atmospheric model uncertainties, and in Ref. [29] the DOC methodology is applied to the powered descent phase of the Mars pinpoint landing problem.

It is expected that the main characteristic of the DOC approach, which is the inclusion of the sensitivity penalty in the performance index, can be extended to the robust filter design problem such that the performance sensitivity of the filters with respect to the model parameter uncertainties can be reduced. Indeed, in previous papers by the authors [30,31] the DOC approach has been successfully incorporated into the Extended Kalman Filter and Unscented Kalman Filter, respectively. The resulting desensitized filters are non-minimum variance but exhibit reduced sensitivity with respect to parameter uncertainties. In some respects, the desensitized filtering approach is similar to the Kalman–Schmidt Filter [5] in that parameter uncertainties are treated in the filter algorithm without the need for dual state-parameter estimation. An important distinction between the two approaches is that the Kalman–Schmidt filter requires that the statistics of the parameter uncertainties are known whereas the new desensitized filtering approach does not.

The purpose of this paper is to extend the desensitized Kalman Filtering approach to the class of Divided Difference Filters [2]. The desensitization of this class of filters follows the same basic methodology developed in [30], which incorporates a weighted norm of the state sensitivities to the minimum-variance cost function used in the DDF. Note that this paper considers only the robustness of the divided difference filtering algorithms to the case of uncertain parameters. Other works such as [32,33] address the robustness of the Divided Difference Filters to the problems of uncertain non-Gaussian noise components.

The remainder of this paper is organized as follows. Section 2 describes the Divided-Difference Filtering algorithms and how the Desensitized Optimal Filtering approach can be applied to develop robust estimators for systems with parameter uncertainty. Specifically, Section 2.1 develops the solution for the First-Order DDF