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Research Article

# Composite fuzzy sliding mode control of nonlinear singularly perturbed systems

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ABSTRACT

This paper deals with the robust asymptotic stabilization for a class of nonlinear singularly perturbed systems using the fuzzy sliding mode control technique. In the proposed approach the original system is decomposed into two subsystems as slow and fast models by the singularly perturbed method. The composite fuzzy sliding mode controller is designed for stabilizing the full order system by combining separately designed slow and fast fuzzy sliding mode controllers. The two-time scale design approach minimizes the effect of boundary layer system on the full order system. A stability analysis allows us to provide sufficient conditions for the asymptotic stability of the full order closed-loop system. The simulation results show improved system performance of the proposed controller as compared to existing methods. The experimentation results validate the effectiveness of the proposed controller.


## 1. Introduction

Whenever a quantitative study of physical systems is undertaken, it is necessary to describe the system in mathematical terms. However, the effort of realistic representation of few physical systems results in higher order dynamics and stiffness of the system. The stiffness difficulties are avoided by modeling the system as a singularly perturbed system. A small singular perturbation parameter $\epsilon$ is exploited to determine the degree of separation between slow and fast subsystems of dynamical systems. The degeneration of the full order system not only eliminates stiffness difficulties, but also makes the boundary layer subsystems asymptotically stable, so that, the deviation rapidly decays. Controlling such systems is a difficult task, as the control has to react with decomposed reduced order slow and fast dynamic states simultaneously. The stability of the full order system can be ensured by the design of separate controllers for subsystems and a combined controller called composite controller applied to govern the full order system [1–3]. The sliding mode control (SMC) is proved to be one of the robust methods to overcome such difficulties. The SMC results in superb system performance which includes insensitivity to parameter variations, and complete disturbance rejection. The control structure of SMC has two control terms, one as continuous control and the other as discontinuous control [4–7].

Several research papers that have made attempts to apply the sliding mode control strategy to singularly perturbed systems have been reported in [8–11]. In [8] a design of SMC for a singular perturbation system with parameter uncertainties and external disturbances was proposed, but it is difficult to compute some parameters for the control law design. Whereas, [9] proposed two-time scale SMC for a nonlinear singularly perturbed system. The full order system was stabilized by combining separately designed SMC controllers for slow and fast subsystems. In [10] global stability of linear time invariant systems was achieved by means of singular perturbations technique and a sliding mode control is synthesized for each subsystem. Recently, in [11] a sliding mode controller for linear singulary perturbed system in the presence of matched bounded external disturbances was proposed. However, these methods suffer from SMC related problems like chattering in control input and state variables. And they are also weak in robustness against parameter variations and disturbances during the reaching phase.

The SMC schemes have been widely applied to control a variety of real world engineering systems [12–17].

Even after sustained active research on SMC, the key problems like chattering, the removal of unmodeled dynamics effects, disturbances and uncertainties during the reaching phase, the need for precise mathematical model of systems and improvement

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of robustness still remains. The chattering effect can be minimized by selecting the smooth approximation like *sat*, *tanh* and continuous control laws [9–11]. Various design techniques have been suggested to improve the performance of SMC for uncertain and chaotic nonlinear systems [18–23]. The practical implementation of SMC is achieved by smoothing the sign function. The controller discontinuities in SMC produce local high-gain controllers. Thus high control forms the linearized closed loop system [11].

To tackle the chattering problem, fuzzy logic controller (FLC) is often used to approximate the discontinuous sign function of SMC. The robust controller is designed by combining the attractive features of SMC and FLC called fuzzy sliding mode control (FSMC). Integration of SMC and FLC methods helps us to overcome the weakness of one method over the other so that guaranteed stability and robust control performance can be achieved. Many researchers have suggested different controls to improve the performance of FSMC using adaptive laws to tune the controller parameters or controller design techniques like two-input single-output (TISO) and fractional order based FSMC [24–30]. Many of these methods are based on the linearized model of the system and may suffer from complex control designs for complex higher order systems.

Only one paper [31] was found on fuzzy sliding mode control for singularly perturbed systems. In [31] a FSMC design method is proposed for a linear singularly perturbed system. In this approach, the fuzzy controller is used as supervisory system to connect two different SMC controllers. The obtained controller is used to stabilize only the slow subsystem by neglecting the dynamics of the fast subsystem. In this approach, the two SMC controllers are designed based on the conventional sliding mode control method. The chattering remains in the control input, the fuzzy controller acts like a supervisory system or an adaptive system for the controller.

This paper investigates the design and application of composite fuzzy sliding mode control for a class of nonlinear singularly perturbed systems. In the proposed method, the original system is decomposed into two reduced order subsystems of different time scales, as slow and fast subsystems, for which stabilizing fuzzy sliding mode control laws are designed separately. The combined controller is called composite fuzzy sliding-mode control applied for the nonlinear singularly perturbed system. The fast subsystem, which represents deviation from the predicted slow behavior, is made asymptotically stable. The fuzzy sliding mode control is designed by defining a switching surface for the system and used as an input to FLC, and using the equivalent control method the continuous control term of SMC is computed. The fuzzy sliding mode control method is used to approximate the discontinuous control term. The stability analysis of the full-order closed-loop system provides sufficient conditions for asymptotic stability and robustness against un-modeled dynamics. The simulation results are compared with fuzzy sliding mode control discussed in [9,11]. To validate the effectiveness of the proposed controller, real time experimentation is carried out on a nonlinear singularly perturbed model of the DC motor and the results are shown with the simulation result. The main contribution of this paper includes the following: (1) a composite fuzzy sliding mode controller is proposed and successfully applied to a class of nonlinear singularly perturbed systems, (2) the advantage of composite fuzzy sliding mode control in reducing the chattering is analyzed and (3) the stability and robustness property is also analyzed.

This paper is organized as follows. Section 2 discusses the problem formulation, in which, we establish composite control design method using sliding mode control technique. Section 3 discusses the proposed composite fuzzy sliding mode control design method and in Section 4, its closed loop stability analysis. The efficacy of the proposed method is illustrated by simulation and experimentation, the comparative results are shown in Section 5. Finally, in Section 6, conclusions are drawn.

## 2. Problem formulation

Consider the following class of nonlinear singularly perturbed system represented in the standard form

$$\dot{x} = f_1(x) + F_2(x)z + b_1(x)u, \quad x(t_0) = x_0, \quad t_0 < \text{or} \geq 0 \tag{1}$$

$$\epsilon\dot{z} = g_1(x) + G_2(x)\, z + b_2(x)u, \quad z(t_0) = z_0 \tag{2}$$

where $x \in R^n$ is the slow state, $z \in R^m$ is the fast state, $u \in R^r$ is the control input, $\epsilon \in (0, 1)$ is the small perturbation parameter and $f_1, g_1$ are the column matrices. $F_2, G_2, b_1$ and $b_2$ are assumed to be continuously differentiable real vector fields. By setting, $\epsilon = 0$ in (2), the systems (1) and (2) degenerate into $n$th order system called a slow subsystem. The slow system is

$$\dot{x}_s = f_1(x_s) + F_2(x_s)z_s + b_1(x_s)u_s \tag{3}$$

$$0 = g_1(x) + G_2(x)z + b_2(x)u, \tag{4}$$

solving (4), we get

$$z_s = h(x_s, u_s) = -G_2^{-1}(x_s)[g_1(x_s) + b_2(x_s)u_s(x_s)] \tag{5}$$

which is a unique root of Eq. (4), substituting for $z_s$ in (3), we get

$$\dot{x}_s = f(x_s) + B(x_s)u_s(x_s), \quad x_s(t_0) = x_0 \tag{6}$$

where

$$f(x_s) = f_1(x_s) - F_2(x_s)G_2^{-1}(x_s)g_1(x_s)$$
$$B(x_s) = b_1(x_s) - F_2(x_s)G_2^{-1}(x_s)b_2(x_s)$$

$x_s$, $z_s$ and $u_s$ denote the slow components of the original variables $x$ and $z$ and (6), is called a slow subsystem from which variable $z$ is excluded by substituting its quasi-steady state value $z_s = h(x_s, u_s)$.

Then the slow manifold can be defined as

$$M_s = [z \in R_s : z = h(x_s, u_s)]$$

the manifold conditions as

$$\epsilon\frac{\partial h}{\partial x}[f_1(x) + F_2(x)z + b_1(x)u] = g_1(x) + G_2(x)z + b_2(x)u \tag{7}$$

must be satisfied for $M_s$ to be an invariant manifold [2]. The fast subsystem or boundary layer system can be derived with the assumption that, the variable $x = constant$ and $\dot{z}_s = 0$ in the boundary layer. Substituting (4) in (2) we get

$$\epsilon\dot{z}_f = G_2(\overline{x})z_f + b_2(\overline{x})u_f, \quad z_f(0) = z_0 - \overline{z}(0) \tag{8}$$

where $z_f = (z - z_s)$ and $u_f = (u - u_s)$.

The variables $t$ and $x$ are slowly varying in foregoing equations, and are given in $\tau$ time scale as

$$t = t_0 + \epsilon\tau, \quad x = x_0(t_0 + \epsilon\tau, \epsilon),$$

setting $\epsilon = 0$ freezes these variables to their initial values ($t = t_0$ and $x = x_0$). The fast subsystem defined on different time scales is derived as

$$\frac{dz_f}{d\tau} = G_2(\overline{x})z_f + b_2(\overline{x})u_f, \quad z_f(0) = z_0 - \overline{z}(0) \tag{9}$$

where $\tau = t/\epsilon$ is the fast time scale. The responses of systems (1) and (2) are approximated under certain conditions as specified in [3], the approximations are

$$x(t) = \overline{x}(t) + O(\mu) \tag{10}$$

$$z(t) = \overline{z}(t) + z_f(t) + O(\mu) \tag{11}$$

Thus, the properties of full order (1) and (2) can be investigated by examining the subsystems (6) and (8).