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Optimal scheduling of multiple sensors in continuous time

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ABSTRACT

This paper considers an optimal sensor scheduling problem in continuous time. In order to make the model more close to the practical problems, suppose that the following conditions are satisfied: only one sensor may be active at any one time; an admissible sensor schedule is a piecewise constant function with a finite number of switches; and each sensor either doesn't operate or operates for a minimum non-negligible amount of time. However, the switching times are unknown, and the feasible region isn't connected. Thus, it's difficult to solve the problem by conventional optimization techniques. To overcome this difficulty, by combining a binary relaxation, a time-scaling transformation and an exact penalty function, an algorithm is developed for solving this problem. Numerical results show that the algorithm is effective.

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1. Introduction

With the development of wireless communication, digital electronics and micro-electromechanical systems, wireless sensor networks have been attracting wide applications, such as pollution monitoring [1], smart grid [2], health care [3] and mobile robotic [4]. However, new challenges in system analysis and design arise due to the unprecedented characteristics [5–7].

Recently, optimal sensor scheduling problems for state estimation have received considerable attention in the control community. The main objective is to minimize the variance of the estimation error, and the motivation arises from resource limitations introduced by wireless sensor networks [8]. In [9], the optimal sensor scheduling problem is modeled in continuous time. Then, the optimal scheduling policy is obtained by solving a quasi-variational inequality. However, the formulation is much too complex. Lee et al. [10] consider the optimal sensor scheduling problem in continuous time, where the control variables are restricted to take values from a discrete set but the switching times are to take place over a continuous time horizon. This formulation leads to an optimal discrete-valued control problem. Ref. [11] discusses the continuous-time optimal sensor scheduling problem by a combination of a branch and cut technique and a gradient-based method. A optimal sensor scheduling problem is considered in [12]. A new heuristic approach, which incorporates the discrete filled function algorithm into standard optimal control software, is proposed for finding a global solution of this problem. For the case of discrete time, the optimal sensor scheduling problem is solved by stochastic strategies [13], the tree search type of algorithms [14], and a combination of a branch and bound and a gradient-based method [15].

In this paper, the optimal sensor scheduling problem in continuous time is reconsidered. Compared with [9–12], the model discussed in the paper is more close to practical problems. However, the switching times are unknown, and the feasible region is not connected. Thus, it is difficult to solve such a problem by standard optimization algorithms, e.g., sequential quadratic programming [17–19], constrained Quasi-Newton method, multiplier penalty function [20], Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm [21], etc. To overcome this difficulty, by combining a binary relaxation, a time-scaling transformation and an exact penalty function, an efficient computational approach is developed for solving this problem. Finally, two numerical examples are provided to illustrate the effectiveness of the developed algorithm.

The rest of the paper is organized as follows. Section 2 presents the optimal scheduling problem of multiple sensors in continuous time. Then, in Section 3, by introducing new binary variables and the time-scaling technique, the sensor optimal scheduling problem is transformed into an equivalent problem with fixed

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switching times. However, the feasible region is not connected, which poses a challenge for standard optimization algorithms. Thus, in Section 4, by an exact penalty function, the problem is transformed into a sequence of unconstrained problems. Each of these unconstrained problems can be effectively solved by any gradient-based optimization technique. Section 5 provides the gradient formulae of the cost function and our algorithm. A numerical example in Section 6 provides evidence that our method is effective.

2. Problem formulation

Consider the optimal sensor scheduling problem in Fig. 1. Let $(\Omega, \mathcal{F}, P)$ be a given probability space. The process is the following stochastic linear system:

$$\dot{x}(t) = A(t)x(t) + B(t)\dot{V}(t), \quad t \in [0, T], \tag{1}$$

$$x(0) = x_0, \tag{2}$$

where $T > 0$ is a given terminal time; for each $t \geq 0$, $A(t) \in R^{n \times n}$ and $B(t) \in R^{n \times p}$ are uniformly bounded measurable matrix-valued functions; x_0 is a R^n-valued Gaussian random vector on $(\Omega, \mathcal{F}, P)$ with mean $E(x_0) = \overline{x}_0$ and covariance $E(x_0 - \overline{x}_0)(x_0 - \overline{x}_0)^{\mathrm{T}} = P_0$; the process $\{V(t), \ t \geq 0\}$ is an R^p-valued Brownian motion on $(\Omega, \mathcal{F}, P)$ with mean $E(V(t)) = 0$ and covariance $E(V(t) - V(s))(V(t) - V(s))^{\mathrm{T}} = Q(t-s)$, where $Q \in R^{p \times p}$ is a symmetric, positive semi-definite matrix; and $\{x(t), \ t \geq 0\}$ is an R^n-valued square integrable process.

Suppose that $x(t)$ can be estimated on the basis of measurement data obtained by N sensors. Then, a sensor schedule can be denoted by a function $u(t) : [0, T] \to \{1, \ldots, N\}$, and $u(t) = i$ indicates that the sensor i is used at time t. Thus, $u(t)$ is completely determined by specifying:

- Switching sequence: the order in which it assumes the different values in $\{1, \ldots, N\}$.
- Switching time: the time at which it switches from one value in $\{1, \ldots, N\}$ to another.

For example, consider an optimal sensor scheduling problem with three sensors and two switches. Suppose that a sensor schedule is defined by

$$u(t) = \begin{cases} 2, & t \in [0, 0.3), \\ 1, & t \in [0.3, 0.7), \\ 3, & t \in [0.7, 1]. \end{cases}$$

Then, (2,1,3) is the switching sequence, 0.3 and 0.7 are the switching time.

In order to make the model more close to the practical problems, suppose that the following three conditions are satisfied:

Assumption 1. In some applications, e.g., robotics, operating several sensors simultaneously causes interference in the system and affects the measurement accuracy [16]. Thus, suppose that only one sensor may be active at any one time.

Assumption 2. Since a sensor schedule with infinitely many switches is not suitable for application in engineering, suppose that $u(t)$ is a piecewise constant function with at most M switches.

Assumption 3. Note that running every potential sensor may not be optimal. Thus, suppose that each sensor either does not operate or operates for a minimum non-negligible amount of time.

These assumptions indicate that adjacent switching times satisfy the following constraint:

$$\tau_i - \tau_{i-1} \in \{0\} \cup [\varepsilon_i, \infty), \tag{3}$$

where τ_i, $i = 1, \ldots, M$, are the ith switching time; and $\tau_0 = 0$, $\tau_{M+1} = T$. Clearly, constraint (3) implies that τ_i, $i = 0, 1, \ldots, M+1$, also satisfy the following conventional ordering constraints:

$$0 = \tau_0 \leq \tau_1 \leq \cdots \leq \tau_{M+1} = T. \tag{4}$$

Clearly, constraint (3) is more complex than conventional constraint (4), which is a simple ordering constraint on the switching times. It must be pointed out that constraint (4) is convex, but constraint (3) is non-convex, and imposing (3) results in a non-connected feasible region for the sensor durations.

Let $\mathcal{U}$ be the set of all such sensor schedules which are measurable, and any $u(t) \in \mathcal{U}$ is referred to as an admissible sensor schedule. Suppose that the state information can be measured using these sensors. Then, for any $u(t) \in \mathcal{U}$, the output equation is given by

$$\dot{y}(t) = \sum_{i=1}^{N} [C_i(t)x(t) + D_i(t)\dot{W}_i(t)]\chi_{\{u(t)=i\}}(t), \quad t \in [0, T], \tag{5}$$

$$y(0) = 0, \tag{6}$$

where for each $t \geq 0$, $C_i(t) \in R^{m \times n}$ and $D_i(t) \in R^{m \times k}$ are uniformly bounded measurable functions; the process $\{W_i(t), \ t \geq 0\}$ is an R^k-valued Brownian motion on $(\Omega, \mathcal{F}, P)$ with mean $E(W_i(t)) = 0$ and covariance $E(W_i(t) - W_i(s))(W_i(t) - W_i(s))^{\mathrm{T}} = R_i(t-s)$, where $R_i \in R^{k \times k}$ is a symmetric, positive definite matrix; and the characteristic function $\chi_{\{u(t)=i\}}(t)$ is defined by

$$\chi_{\{u(t)=i\}}(t) = \begin{cases} 1 & \text{if } u(t) = i, \\ 0 & \text{if } u(t) \neq i. \end{cases} \tag{7}$$

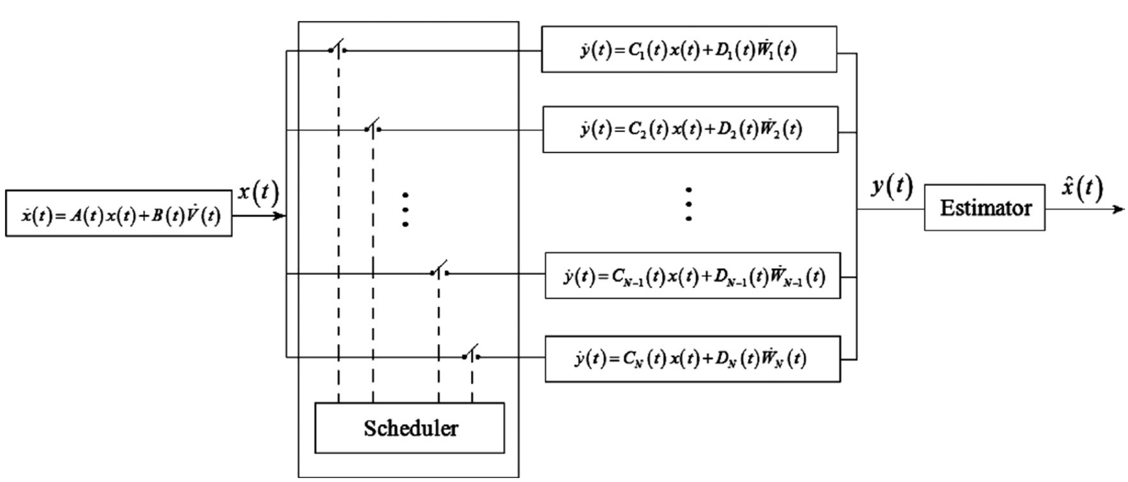

Fig. 1. Block diagram of optimal scheduling of multiple sensors in continuous time.

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