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Research Article

# Robust fault-tolerant tracking control design for spacecraft under control input saturation

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ABSTRACT

In this paper, a continuous globally stable tracking control algorithm is proposed for a spacecraft in the presence of unknown actuator failure, control input saturation, uncertainty in inertial matrix and external disturbances. The design method is based on variable structure control and has the following properties: (1) fast and accurate response in the presence of bounded disturbances; (2) robust to the partial loss of actuator effectiveness; (3) explicit consideration of control input saturation; and (4) robust to uncertainty in inertial matrix. In contrast to traditional fault-tolerant control methods, the proposed controller does not require knowledge of the actuator faults and is implemented without explicit fault detection and isolation processes. In the proposed controller a single parameter is adjusted dynamically in such a way that it is possible to prove that both attitude and angular velocity errors will tend to zero asymptotically. The stability proof is based on a Lyapunov analysis and the properties of the singularity free quaternion representation of spacecraft dynamics. Results of numerical simulations state that the proposed controller is successful in achieving high attitude performance in the presence of external disturbances, actuator failures, and control input saturation.



## 1. Introduction

One of the challenging problems in the field of aerospace engineering is designing a spacecraft attitude tracking controller to maintain stability and performance in the presence of actuator failures, external disturbances, uncertainty in inertial matrix and control input saturation.

A conventional feedback control design for a complex system may result in an unsatisfactory performance, or even instability, in the event of malfunctions in actuators, sensors or other system components. To overcome such weaknesses, new approaches to control system design have been developed in order to tolerate component malfunctions while maintaining desirable stability and performance properties. These types of control systems are often known as fault-tolerant control systems (FTCS). Generally speaking, FTCS can be classified into two types: passive (PFTCS) and active (AFTCS). In PFTCS, controllers are fixed and are designed to be robust against a class of presumed faults. This approach needs neither fault detection and isolation (FDI) schemes nor controller reconfiguration. Compared to the passive approach, the active FTC approach requires a FDI mechanism to detect and identify the faults in real time, and then a mechanism to reconfigure the controllers according to the online fault information from the FDI. Compared to the passive approach, the AFTCS need significantly more computational power to implement. Furthermore, there is a time delay between the detection of faults and the reconfiguration of the controller in this approach [1]. These drawbacks motivate us for the investigation of a passive fault-tolerant controller for a spacecraft attitude control system with the occurrence of unexpected faults.

Numerous research results are available for the passive fault-tolerant controller design with different approaches, such as linear matrix inequalities (LMIs) schemes [2], $H_\infty$ [3], adaptive control [4], sliding mode control [5], fuzzy logic [6] and neural networks [7]. Authors in [8] proposed four different controllers for certain and uncertain plants based on the absolute stability theory to handle the loss of actuator effectiveness; however, the designed controllers performed unsatisfactorily for systems with actuator failures [9].

In [2] a reliable robust fault-tolerant controller based on an LMI approach is designed. Model matching is the method used in [10] for actuator fault tolerant control and in [11] a robust fault tolerant control which is capable of attenuating both bounded and unbounded disturbances is proposed. In [12], performance indices are explicitly considered while stabilizing the attitude control of

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spacecraft. However, most of these controllers can only be applied to linear systems and are not applicable for non-linear dynamics.

Although in [13–16] a range of controllers to effectively handle the limited actuator output have been developed, actuator failures have not been taken into account in them. Considering actuator failures, there have been a number of results in the literature on attitude control of the spacecraft [7,17–22]. However, the issue of control input constraints has not been dealt within these approaches. Also Refs. [23–25] considered these two issues concurrently, but the disturbance effect is not taken into account.

To deal with the problem of actuator failures in presence of actuator saturation and external disturbances, Ref. [9] developed a robust fault-tolerant controller for spacecraft attitude control subsystem. But the proposed method fails in the situation in which any maneuver is required. Also robustness to uncertain inertial matrix is not considered.

To achieve high attitude performance, several issues including external disturbances, actuator failures, uncertainty in inertial matrix and control input saturation are required to be explicitly taken into account in the attitude controller design, which makes the controller design much more difficult.

To address this problem, a robust fault tolerant attitude tracking controller based on the variable structure approach is proposed for attitude control of the spacecraft with explicit consideration of external disturbances, actuator failures, and control input saturation. A key feature of the proposed strategy is that the design of the FTC is independent of the information about the faults.

Also the unit quaternion is employed to describe the attitude of a rigid spacecraft because of its global representation without singularities. The asymptotic stability of the closed-loop system is guaranteed by the Lyapunov direct approach and numerical simulations are carried out on the governing non-linear system equations of motion to show the performance of the proposed controller.

This paper is organized as follows. In Section 2, the spacecraft attitude dynamics are introduced. Fault-tolerant controllers are derived in Section 3. The results of numerical simulations are presented in Section 4. Finally, the paper is completed with some concluding remarks.

## 2. System model and equations of motion

### 2.1. Spacecraft attitude dynamics

The spacecraft is modeled as a rigid body with actuators that provide torques about three mutually perpendicular axes that defines a body-fixed frame ($\mathbf{B}$). The equations of motions are given by [26]

$$\dot{q}_0 = -\frac{1}{2}\mathbf{q}^T\boldsymbol{\omega} \tag{1}$$

$$\dot{\mathbf{q}} = \frac{1}{2}(\mathbf{q}^\times + q_0\mathbf{I}_3)\boldsymbol{\omega} \tag{2}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} = \mathbf{u} + \mathbf{d} \tag{3}$$

where $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^T$ is the spacecraft angular velocity with respect to an inertial frame ($\mathbf{I}$) and expressed in the body-fixed frame ($\mathbf{B}$), the unit quaternion $\mathbf{Q} = (q_0, \mathbf{q}^T) \in \mathbb{R} \times \mathbb{R}^3$ describes the attitude orientation of the spacecraft in ($\mathbf{B}$) with respect to ($\mathbf{I}$), and satisfies $\mathbf{q}^T\mathbf{q} + q_0^2 = 1$, $\mathbf{I}_3$ denotes the $3 \times 3$ identity matrix, $\mathbf{J} \in \mathbb{R}^{3\times3}$ represents the positive definite spacecraft inertial matrix which has the property $J_m\|\mathbf{x}\|^2 \le \mathbf{x}^T\mathbf{J}\mathbf{x} \le J_M\|\mathbf{x}\|^2, \quad \forall \mathbf{x} \in \mathbb{R}^3$ where $J_m$ and $J_M$ are the positive constants, $\mathbf{u} = (u_1, u_2, u_3)^T \in \mathbb{R}^3$ is the control torque input generated by actuators, and $\mathbf{d} = (d_1, d_2, d_3)^T \in \mathbb{R}^3$ denotes the disturbance torque. For, $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)^T$ the notation $\boldsymbol{\xi}^\times$ denotes the following skew symmetric matrix:

$$\boldsymbol{\xi}^\times = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix} \tag{4}$$

**Remark 1.** Eqs. (1)–(3) are the dynamics equations of a rigid spacecraft equipped with thrusters as attitude control actuators and star tracker as attitude determination sensor.

For the development of the control laws, the following assumptions are made:

**Assumption 1.** All three components of the control torque, $\mathbf{u}$, are constrained by a bounded value, expressed by

$$|u_i| \le u_{\max}\, \forall t > 0 \quad i = 1, 2, 3 \tag{5}$$

**Assumption 2.** The disturbance, $\mathbf{d}$, is bounded, and for all elements of $d_i$ there exists a positive but known constant, $\overline{d}$ such that $|d_i| \le \overline{d}$.

### 2.2. Desired dynamics

The desired motion of the spacecraft is specified by the attitude of a frame ($\mathbf{D}$) whose orientation with respect to ($\mathbf{I}$) is described by the unit quaternion $\mathbf{Q}_d = (q_{0d}, \mathbf{q}_d^T) \in \mathbb{R} \times \mathbb{R}^3$ that satisfy the constraint $\mathbf{q}_d^T\mathbf{q}_d + q_{0d}^2 = 1$. Let $\boldsymbol{\omega}_d = (\omega_{d1}, \omega_{d2}, \omega_{d3})^T$ denote the angular velocity of ($\mathbf{D}$) with respect to ($\mathbf{I}$), which is equivalent to the desired angular velocity of the spacecraft expressed in the frame ($\mathbf{D}$). The following assumption is made about $\boldsymbol{\omega}_d$ and $\dot{\boldsymbol{\omega}}_d$:

**Assumption 3.** There exist constants $\overline{\boldsymbol{\omega}}_d \ge 0$ and $\overline{\dot{\boldsymbol{\omega}}}_d \ge 0$, such that $|\omega_{di}| \le \overline{\omega}_d, \quad i = 1, 2, 3$ and $|\dot{\omega}_{di}| \le \overline{\dot{\omega}}_d, \quad i = 1, 2, 3$ for all $t \ge 0$.

### 2.3. Spacecraft attitude error dynamics

To address the attitude tracking problem, the attitude tracking error $\mathbf{Q}_e = (q_{0_e}, \mathbf{q}_e^T)^T$ is defined as the relative orientation between the body frame ($\mathbf{B}$) and the desired frame ($\mathbf{D}$) and it is computed by the quaternion multiplication rule [26] as

$$\mathbf{q}_e = q_{0_d}\mathbf{q} - q_0\mathbf{q}_d + \mathbf{q}^\times\mathbf{q}_d \tag{6}$$

$$q_{0_e} = q_{0_d}q_0 + \mathbf{q}_d^T\mathbf{q} \tag{7}$$

The corresponding rotation matrix is given by

$$\mathbf{C}(\mathbf{Q}_e) = (q_{0_e}^2 - \mathbf{q}_e^T\mathbf{q}_e)\mathbf{I}_3 + 2\mathbf{q}_e\mathbf{q}_e^T - 2q_{0_e}\mathbf{q}_e^\times \tag{8}$$

Note that $\|\mathbf{C}\| = 1$ and $\dot{\mathbf{C}} = -\boldsymbol{\omega}_e^\times\mathbf{C}$, where the relative angular velocity $\boldsymbol{\omega}_e$ of ($\mathbf{B}$) with respect to ($\mathbf{D}$) is defined as $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \mathbf{C}\boldsymbol{\omega}_d$.

Now, the governing differential equations for the attitude tracking error, $\mathbf{q}_e$, are stated as follows:

$$\dot{q}_{0_e} = -\tfrac{1}{2}\mathbf{q}_e^T\boldsymbol{\omega}_e \tag{9}$$

$$\dot{\mathbf{q}}_e = \tfrac{1}{2}(\mathbf{q}_e^\times + q_{0_e}\mathbf{I}_3)\boldsymbol{\omega}_e \tag{10}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} + \mathbf{u} + \mathbf{d} - \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d - \boldsymbol{\omega}_e^\times\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d) \tag{11}$$

When the spacecraft has three actuators and some of them partially fail, the attitude dynamics of the spacecraft is expressed as

$$\mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} + \boldsymbol{\Gamma}\mathbf{u} + \mathbf{d} - \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d - \boldsymbol{\omega}_e^\times\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d) \tag{12}$$

where $\boldsymbol{\Gamma} = \mathrm{diag}\{\Gamma_1, \Gamma_2, \Gamma_3\}$, $0 < \Gamma_i \le 1$ is the actuation effectiveness matrix. The case in which $\Gamma_i = 1$ implies that the $i$th actuator is healthy, and $0 < \Gamma_i < 1$ corresponds to the case in which the $i$th actuator partially fails.