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ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

A novel approach for analyzing the behavior of industrial systems using weakest t-norm and intuitionistic fuzzy set theory

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ARTICLE INFO

Article history:
Received 1 November 2013
Received in revised form
14 March 2014
Accepted 17 March 2014
This paper was recommended for publication by Dr. A.B. Rad.

Keywords:
Reliability
Arithmetic operation
Weakest t-norm
Membership function
Repairable industrial system
Intuitionistic fuzzy set

ABSTRACT

The present work investigates the various reliability parameters of industrial systems in terms of membership and non-membership functions by using α-cut and the weakest t-norm based arithmetic operations on triangular intuitionistic fuzzy sets. As the available information about the constituent components of the system is most of the time imprecise, incomplete, vague and conflicting, the management decisions are based on experience. Thus, the objective of this paper is to quantify the uncertainties that make the decisions realistic, generic and extensible for the application domain. Sensitivity of system performance has also been analyzed for showing the effect of taking wrong combinations of reliability parameters. The obtained results computed by the proposed approach are compared with the existing methodologies. The approach has been illustrated through a case study for supremacy.

1. Introduction

The development of science and technology and the needs of modern society are racing against each other. Industries are trying to introduce more and more automation in their industrial processes in order to meet the ever-increasing demands of society. The complexity of industrial systems as well as their products is increasing day-by-day. Due to this, the job of the system analyst and plant personnel becomes so challenging in order to maintain the profile of the system so that it becomes operating continuously for a longer time. On the other hand, failure is an inevitable phenomenon for all industrial systems. These failures may be the results of human error, poor maintenance or inadequate testing/ inspection which affect each part/unit of the system differently, and thus the issue is subject to uncertainty. To this effect, both probabilistic and non-probabilistic methods are used to treat the element of uncertainty in the analysis. Although the probability approach has been applied successfully for many real world engineering problems still there are some limitations to the probabilistic method. For instance, probabilistic methods are based on mass collection of data, which is random in nature, to achieve the requisite confidence level. But in a large scale the complicated system has the massive fuzzy uncertainty due to which it is difficult to get the exact probability of the events. Also, at early stages of new product development, the available data (numbers of testing samples, recorded failures on test) is limited, so the required confidence level may not be met if probability methods are used. Thus, due to these limitations, the results based on probability theory do not always provide useful information to the practitioners and hence the probabilistic approach to the conventional reliability analysis is inadequate to account for such in-built uncertainties in the data. To overcome these difficulties, Zadeh [1] proposed the concept of fuzzy sets and established fuzzy set theory, which formed the foundation for describing and processing uncertain information. After that, a lot of progress has been made in both theory and application. The theory of fuzzy numbers, an important part of fuzzy set theory, is very popular in describing uncertain phenomena in actual problems. Fuzzy arithmetic, following Zadeh's extension principle [1] in fuzzy set theory, was investigated by various researchers which include, in the approximate and exact manners [2,3], t-norm operators [4,5], and the weakest t-norm (T_ω) operator [6–9], and others. As far as the reliability is concerned, various researchers [10–15] have used fuzzy set theory and fuzzy arithmetic during their evaluation of system reliability parameters. The major disadvantages of their methodologies are that they do not consider the degree of hesitation between the membership functions, i.e. in their analysis there is a zero degree of hesitation between the membership functions of the element. Also the level of confidence domain of the expert is taken as one. But in most of the real life modeling,

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http://dx.doi.org/10.1016/j.isatra.2014.03.014
0019-0578/

Please cite this article as: Garg H. A novel approach for analyzing the behavior of industrial systems using weakest t-norm and intuitionistic fuzzy set theory. ISA Transactions (2014), http://dx.doi.org/10.1016/j.isatra.2014.03.014

there is always a degree of hesitation between the membership functions and their corresponding domain level is less than one. Therefore, there is a need for a suitable methodology which is applied for analyzing their parameters, by utilizing uncertain data, in a more realistic way. Additionally, there are some other types of reliability problems developed by the researchers such as process control [16] reliability, distribution system reliability [17], and reliability of dynamic systems [18].

After the introduction of the concept of fuzzy sets, several researches were conducted on the extensions of the notion of fuzzy sets. The theory of intuitionistic fuzzy sets (IFSs), first proposed by Attanassov [19], is an extension of traditional fuzzy sets, whose basic component is only a membership function. IFSs are described by two characteristic functions expressing the degrees of membership and non-membership of elements in the universe, considered as a generalization of the ordinary fuzzy sets for conditions with insufficient information to define an imprecise concept by traditional fuzzy sets. Although the concept of vague sets was presented in a later work [20], they are considered the equivalent mathematical form as IFSs [21]. Chen [22,23] presented the arithmetic operations and analyzing the fuzzy system reliability based on vague sets. Shu et al. [24] used IFS for failure analysis of the problem of a printed circuit board assembly. Chang et al. [25] proposed a vague fault-tree analysis procedure to determine the weapon system's reliability. Kumar et al. [26] presented a method for fuzzy system reliability analysis using the idea of interval valued vague sets and intuitionistic fuzzy numbers respectively. Recently, Garg et al. [27] presented an intuitionistic fuzzy optimization technique for solving the multi-objective reliability optimization problems in an interval environment. Apart from that a lot of work has been done to develop and enrich the IFS theory given in [28–31] and their corresponding references in terms of reliability evaluation of series–parallel system. As all the above researchers have analyzed the system reliability only for measuring the performance of the system but in real-life modeling other reliability indices namely failure rate, repair time, mean time between failures (MTBF), etc. also affect the system performance. Thus it is necessary to analyze all these parameters which affect the systems' performance and consequently their behavior. To remove this, Garg [32] analyzed the various reliability parameters namely systems' failure rate, repair time, MTBF, expected number of failures (ENOF), reliability, availability and maintainability in terms of intuitionistic fuzzy membership functions. Intuitionistic fuzzy numbers (IFNs) are taken by him to handle the uncertainties in the data and modeled the system with the help of Petri nets. If the above techniques have been used for computing the system reliability parameters in the form of membership functions then there exists a wide range of uncertainty in the analysis in the form of spread due to various arithmetic operations during the calculation. Thus for handling this issue, an efficient technique is needed which will reduce the uncertainty level, in the form of spread, during the analysis and hence gave more sound decision for the system analyst for analyzing the behavior of the system in a lesser time. Therefore, in the present work, an investigation has been done for quantifying the uncertainties which are present in the data so as to make the decisions realistic, generic and extensible for the application domain. For this, the uncertainties which are present in the data are handled with the help defining their corresponding intuitionistic fuzzy numbers by taking different levels of spread, say $\pm 15\%$, $\pm 25\%$, $\pm 50\%$ on both sides of the input (crisp) data. After quantifying the uncertainties, weakest t-norm based arithmetic operations have been used, instead of fuzzy arithmetic operations, for computing the system's parameters for increasing the relevance of the study.

Thus the main objective of this paper is to present a methodology for analyzing the behavior of the repairable industrial systems by utilizing limited and vague data. In the analysis, uncertainties which are presented in the data are handled with the help of defining the IFNs corresponding to different levels of spreads, and the weakest t-norm operations have been used for finding the system parameters' expression. The modeling of the system is done with the help of fault tree analysis. Various reliability parameters for strengthening the analysis have been done in terms of membership and non-membership functions by considering the degree of confidence level (α). Sensitivity analysis on the system MTBF are done for showing the effectiveness of the proposed approach. Finally, the computed results are compared with the existing methodology results and gave a recommendation for improving the production as well as productivity of the system. The approach has been demonstrated through a case study of the washing unit in a paper mill, a repairable complex industrial system.

The rest of the paper is organized as follows: Section 2 define the basic definitions of intuitionistic fuzzy set theory along with the weakest t-norm and their corresponding arithmetic operations. Section 3 describes the methodology for analyzing the behavior of industrial system along with the existing methodologies while the approach has been illustrated through a case study whose description is given in Section 4. The results and discussion of these problems have been presented in Section 5. Finally, some concrete conclusion and future research directions drawn are presented in Section 6.

2. Preliminaries

2.1. Intuitionistic fuzzy set [19]

Let X be a fixed set, then $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\}$ is called an intuitionistic fuzzy set [19], where the functions $\mu_{\tilde{A}} : X \to [0,1]$ and $\nu_{\tilde{A}} : X \to [0,1]$ denote the degree of membership and non-membership of the element $x \in X$ to $\tilde{A}$ respectively for every $x \in X$ such that $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$. In addition $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$ is called the degree of hesitation or uncertainty of x to $\tilde{A}$. If $\pi_{\tilde{A}}(x) = 0$ then intuitionistic fuzzy set $\tilde{A}$ is reduced to a fuzzy set.

2.2. Convex intuitionistic fuzzy set [19]

An IFS $\tilde{A}$ in universe X is convex if and only if

(i) Membership functions of $\mu_{\tilde{A}}(x)$ of $\tilde{A}$ is fuzzy – convex i.e.

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in X, 0 \le \lambda \le 1 \tag{1}$$

(ii) Non-membership functions of $\nu_{\tilde{A}}(x)$ of $\tilde{A}$ is fuzzy – concave i.e.

$$\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \le \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in X, 0 \le \lambda \le 1 \tag{2}$$

2.3. Intuitionistic fuzzy number

An IFS $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in \mathbb{R}\}$ such that $\mu_{\tilde{A}}$ and $1-\nu_{\tilde{A}}$, where

$$(1-\nu_{\tilde{A}})(x) = 1 - \nu_{\tilde{A}}(x), \quad \forall x \in U$$

are fuzzy numbers, is called an intuitionistic fuzzy number (IFN).

2.4. α-cut of an IFS

The α-level set of an IFS $\tilde{A} : \mathbb{R} \to [0,1]$, $\alpha \in [0,1]$, denoted by $A^{(\alpha)}$ is defined as a crisp set which consists of elements of $\tilde{A}$ having at