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Comput. Methods Appl. Mech. Engrg. 195 (2006) 5297–5315

Computer methods in applied mechanics and engineering

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Rotation-free finite element for the non-linear analysis of beams and axisymmetric shells

Fernando G. Flores [a,*], Eugenio Oñate [b]

[a] National University of Córdoba, Casilla de Correo 916, 5000 Córdoba, Argentina
[b] International Centre for Numerical Methods in Engineering, Edificio C1, Gran Capitán s/n, 08034 Barcelona, Spain

Received 8 March 2005; received in revised form 29 August 2005; accepted 30 August 2005

Abstract

In this paper a finite element for the non-linear analysis of two-dimensional beams and axisymmetric shells is presented. The element uses classical thin shell assumptions (no transverse shear strains). The main feature of the element is that it has no rotational degrees of freedom. Curvatures are computed using geometrical information from the patch of three elements formed by the main element and the two neighbor (adjacent) elements. Special attention is devoted to non-smooth geometries and branching shells. An elastic–plastic material law is considered. Large strains are treated using a logarithmic strain measure and a through-the-thickness numerical integration of the constitutive equations. Several examples are presented including linear problems to study convergence properties, and non-linear problems for both elastic and elastic–plastic materials and large strains.

Keywords: Finite elements; Beams; Axisymmetric shells; Rotation-free element; Large strains

1. Introduction

The development of numerical techniques for shell analysis without rotational degrees of freedom has been mainly associated to the finite differences method (see, for example, Refs. [1–3]). Nevertheless, the idea of developing finite elements without rotational DOFs is not new [4,5] and different attempts have been reported in the last 20 years [6–10]. Despite these attempts it is just in the last few years that reliable rotation-free elements for industrial applications have been developed [11–14]. All the approximations share in common the definition of a patch (neighborhood) of elements to interpolate the geometry and the displacements. The main difference between the different approaches is the interpolation of the geometry and the theoretical basis used. One of the main aspects that remain unsolved satisfactorily is the treatment of non-smooth surfaces and specially branching shells. An adequate handling of multiple surfaces, i.e., when more than two elements share a side or edge, is mandatory if the element is to be used for the analysis of aeronautic and naval structures or frame structures typical in buildings, among others.

In this paper, two-dimensional shell problems are tackled (previous work in this line can be found in Refs. [15,16]) with special focus on non-smooth shells and branching lines. This work may be seen as a first step towards the development of three-dimensional rotation-free shell finite elements capable of handling kinked and branching shells.

The outline of the paper is as follows. First, a summary of the most relevant equations governing the kinematic behavior of two-dimensional Kirchhoff–Love shells is presented. In Section 3, a two-node rotation-free finite element for the analysis

* Corresponding author.
E-mail address: fflores@efn.uncor.edu (F.G. Flores).

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doi:10.1016/j.cma.2005.08.021

of straight but non-homogeneous (different thickness or materials) beam/axisymmetric shells is developed. In Section 4, the element is extended to deal with curved and non-smooth shells. The formulation is extended further to branching shells in Section 5. Several examples are presented in Section 6 showing convergence properties in linear and non-linear problems. At the end of this section, two examples including elasto-plasticity with large strains are shown. In Section 7, some conclusions are drawn.

2. Governing equations of beams and thin two-dimensional shells

In this section a summary of the most relevant equations governing the kinematic behavior of thin shells is presented. More details can be found in the wide literature dedicated to the subject [1]. In order to introduce the problem, the kinematics of two-dimensional beams are considered. These equations are latter extended to axisymmetric shells.

2.1. Euler–Bernoulli beams

Within a classical Euler–Bernoulli beam theory (transverse shear strains disregarded) the geometry of a curved beam (see Fig. 1) is completely defined by the position of central axis (line of centroids) $\boldsymbol{\varphi}$ as a function of the arc length s measured from an arbitrary point on the reference (undeformed) configuration

$$\boldsymbol{\varphi}(s) = \begin{bmatrix} \varphi_1(s) \\ \varphi_2(s) \end{bmatrix}, \tag{1}$$

where φ_i are the Cartesian components referred to the canonical base $(\mathbf{e}_1, \mathbf{e}_2)$. At each material point of the line of centroids associated to the coordinate s it is possible to compute a convective system defined by the tangent $\mathbf{t}$ to the line of centroids $\boldsymbol{\varphi}$ and the normal (in the plane) $\mathbf{n}$

$$\mathbf{t}(s) = \frac{\dfrac{\mathrm{d}\boldsymbol{\varphi}(s)}{\mathrm{d}s}}{\left\| \dfrac{\mathrm{d}\boldsymbol{\varphi}(s)}{\mathrm{d}s} \right\|}, \qquad \mathbf{n}(s) = \mathbf{e}_3 \times \mathbf{t}(s) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{t}(s), \tag{2}$$

where $\mathbf{e}_3$ is the normal to the beam plane.

The position $\mathbf{x}$ of an arbitrary point located on the beam cross section can be written in terms of the position of the central line $\boldsymbol{\varphi}$ and the normal direction $\mathbf{n}$ as a function of the distance ζ of the point to the central axis as

$$\mathbf{x}(s,\zeta) = \begin{bmatrix} x_1(s,\zeta) \\ x_2(s,\zeta) \end{bmatrix} = \boldsymbol{\varphi}(s) + \zeta \mathbf{n}(s). \tag{3}$$

The undeformed stress-free (natural) configuration will be used as the reference configuration where the material points are defined by their coordinates (s and ζ). Values measured on the reference configuration will be denoted with a left supra-index "o".

Dropping in the notation the dependence with the arc length s, and denoting by $\frac{\partial(\tilde{\ })}{\partial s} = (\tilde{\ })_{/s}$, the in-plane deformation gradient is defined by

$$\mathbf{F}(s,\zeta) = \left[\frac{\partial \mathbf{x}}{\partial s}, \quad \frac{\partial \mathbf{x}}{\partial \zeta} \right] = [\boldsymbol{\varphi}_{/s} + \zeta \mathbf{n}_{/s}, \quad \mathbf{n}]. \tag{4}$$

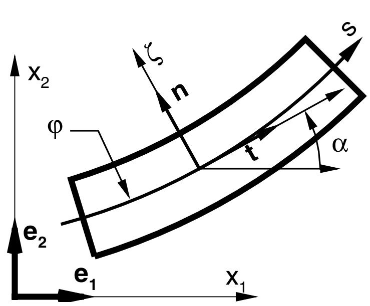

Fig. 1. Beam geometry and notation.