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Research Article

New results on stability analysis for time-varying delay systems with non-linear perturbations

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ABSTRACT

The problem of stability for linear time-varying delay systems under nonlinear perturbation is discussed, with delay assumed as time-varying. Delay decomposition approach allows information of the delayed plant states to be fully considered. A less conservative delay-dependent robust stability condition is derived, using integral inequality approach to express the relationship of Leibniz–Newton formula terms in the within the framework of linear matrix inequalities (LMIs). Merits of the proposed results lie in lesser conservatism, which are realized by choosing different Lyapunov matrices in the decomposed integral intervals and estimating the upper bound of some cross term more exactly. Numerical examples are given to illustrate the effectiveness and lesser conservatism of the proposed method.

1. Introduction

Delay phenomenon is often encountered in various mechanics, physics, biology, medicine, economy, and engineering systems, such as AIDS epidemic, aircraft stabilization, chemical engineering systems, control of epidemics, distributed networks, inferred grinding model manual control, microwave oscillator, models of lasers, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines [1–10,13–17,20–24]. Stability analysis of dynamic systems with time-delay is thus the focus of theoretical and practical importance, with many researchers recently paying heed to delay-dependent stability criteria, generally less conservative than delay-independent ones [1,2,4–10,13–16,21–24]. In practice, systems almost always present uncertainty, since it is very difficult to obtain an exact mathematical model due to environmental noise, uncertain or slowly varying parameters, etc. Considerable efforts have been devoted to stability for time-delay systems with nonlinear perturbation [2,4–9,12–16,21,22] and references therein. Fuzzy control methodologies have emerged in recent years as promising ways to approach nonlinear control problems. Fuzzy control, in particular, has had an impact in the control community because of the simple approach it provides to use heuristic control knowledge for nonlinear control problem. In very complicated situations, where the plant parameters are subject to perturbations or when the dynamics of the systems are too complex for a mathematical model to describe, adaptive schemes have to be used online to gather data and adjust the control parameters automatically [3,11,12,17–20].

To derive a less conservative stability criterion, model transformation was used in [2] and parameterized neutral model transformation utilized in [14,15]. In [5], employing descriptor model transformation, delay-dependent robust stability condition was presented for a class of time-delay systems with nonlinear perturbation. In [22], using free-weighting matrices to deal with cross terms involved in the derivative of the Lyapunov–Krasovskii function, a less conservative delay-dependent stability criterion was proposed. Subsequent analysis yields convex LMI condition, non-conservatively solved at boundary conditions. Despite using slack matrices in delay-dependent analysis, total number of decision variables involved in the proposed LMI criterion is less than that of [21], rendering criterion not only less conservative but also computationally more attractive. Jensen integral inequality approach was taken in [23,24]. In [9], Liu proposed integral inequality approach to rate the delay-dependent stability; a less conservative delay-dependent stability criterion was provided in [23,24] via delay decomposition approach. On the other hand, choosing an appropriate Lyapunov–Krasovskii function and estimating the upper bound of its time derivative is very important in deriving the stability criteria.

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Motivated by the afore-mentioned analysis, this paper deals with delay-dependent stability for a class of time-varying delay systems with nonlinear perturbations. By developing delay decomposition approach, information of delayed plant states can be taken into full consideration, new delay-dependent sufficient stability criteria obtained in terms of linear matrix inequalities. Merits of the proposed results lie in their less conservatism and are realized by choosing diverse Lyapunov matrices in decomposed integral intervals and estimating the upper bound of some cross term more exactly. Proposed stability criteria are formulated in terms of a set of linear matrix inequalities (LMIs). Finally, two numerical examples show efficacy of the proposed approach.

2. Main results

This paper considers time-varying delay systems with nonlinear perturbations that can be described by linear differential difference equations:

$$\dot{x}(t)=Ax(t)+Bx(t-h(t))+Ff(x(t),t)+Gg(x(t-h(t)),t)\ \mathrm{t}>0 \tag{1}$$

$$x(t+\eta)=\phi(\eta),\quad \forall\eta\in[-h,\ 0] \tag{2}$$

with $x(t)\in R^n$ as state vector of the system, $A,B,F,G\in R^{n\times n}$ constant matrices, $\phi(\bullet)$ continuous vector-valued initial function, $h(t)$ a time-varying delay in the state, h an upper bound on delay $h(t)$. $f(x(t),t)\in R^n$, and $g(x(t-h(t)),t)\in R^n$ unknown non-linear perturbations with respect to $x(t)$ and $x(t-h(t))$, respectively, assumed as

$$f^T(x(t),t)f(x(t),t)\le\alpha^2x^T(t)x(t) \tag{3}$$

$$g^T(x(t-h(t)),t)g(x(t-h(t)),t)\le\beta^2x^T(t-h(t))x(t-h(t)) \tag{4}$$

where α and β are known positive constants.

We consider two different cases for time-varying delays

Case I. $h(t)$ is a differentiable function, satisfying for all $t\ge0$:

$$0\le h(t)\le h\ \text{ and }\ \dot{h}(t)\le h_d \tag{5}$$

Case II. $h(t)$ is not differentiable or upper bound of $h(t)$ derivative unknown, and $h(t)$ satisfies

$$0\le h(t)\le h \tag{6}$$

where h and h_d are some positive constants.

The following lemma is useful in deriving criteria:

Lemma 1. [9,10] For any positive semi-definite matrices

$$X=\begin{bmatrix}X_{11}&X_{12}&X_{13}\\X_{12}^T&X_{22}&X_{23}\\X_{13}^T&X_{23}^T&X_{33}\end{bmatrix}\ge0 \tag{7}$$

the following integral inequality holds

$$-\int_{t-h(t)}^t\dot{x}^T(s)X_{33}\dot{x}(s)ds\le$$
$$\int_{t-h(t)}^t\begin{bmatrix}x^T(t)&x^T(t-h(t))&\dot{x}^T(s)\end{bmatrix}\begin{bmatrix}X_{11}&X_{12}&X_{13}\\X_{12}^T&X_{22}&X_{23}\\X_{13}^T&X_{23}^T&0\end{bmatrix}\begin{bmatrix}x(t)\\x(t-h(t))\\\dot{x}(s)\end{bmatrix}ds \tag{8}$$

For system (1)–(6) we give stability condition via delay decomposition approach:

Theorem 1. In Cases I, if $0\le h(t)\le\delta h$, for given three scalars h, δ, and h_d. Then, for any delay $h(t)$ satisfy $0\le h(t)\le h$, $\dot{h}(t)\le h_d$, and $0<\delta<1$, the system described by (1) with (5) is asymptotically stable if there exist matrices $P=P^T>0$, $Q_i=Q_i^T>0$, $R_i=R_i^T>0$, $(i=1,2,3)$, and positive semi-definite matrices

$$X=\begin{bmatrix}X_{11}&X_{12}&X_{13}\\X_{12}^T&X_{22}&X_{23}\\X_{13}^T&X_{23}^T&X_{33}\end{bmatrix}\ge0,$$
$$Y=\begin{bmatrix}Y_{11}&Y_{12}&Y_{13}\\Y_{12}^T&Y_{22}&Y_{23}\\Y_{13}^T&Y_{23}^T&Y_{33}\end{bmatrix}\ge0,$$
$$Z=\begin{bmatrix}Z_{11}&Z_{12}&Z_{13}\\Z_{12}^T&Z_{22}&Z_{23}\\Z_{13}^T&Z_{23}^T&Z_{33}\end{bmatrix}\ge0$$

such that

$$\Omega=\begin{bmatrix}\Omega_{11}&\Omega_{12}&\Omega_{13}&\Omega_{14}&0&0&\Omega_{17}\\\Omega_{12}^T&\Omega_{22}&0&0&\Omega_{25}&0&\Omega_{27}\\\Omega_{13}^T&0&\Omega_{33}&0&0&0&\Omega_{37}\\\Omega_{14}^T&0&0&\Omega_{44}&0&0&\Omega_{47}\\0&\Omega_{25}^T&0&0&\Omega_{55}&\Omega_{56}&0\\0&0&0&0&\Omega_{56}^T&\Omega_{66}&0\\\Omega_{17}^T&\Omega_{27}^T&\Omega_{37}^T&\Omega_{47}^T&0&0&\Omega_{77}\end{bmatrix}<0 \tag{9}$$

and

$$R_1-X_{33}\ge0,\ R_2-Y_{33}\ge0,\ R_1+(1-h_d)R_3-Z_{33}\ge0 \tag{10}$$

where

$\Omega_{11}=A^TP+PA+Q_1+Q_3+\varepsilon_1\alpha^2I+\delta hZ_{11}+Z_{13}+Z_{13}^T$,
$\Omega_{12}=PB+\delta hZ_{12}-Z_{13}+Z_{23}^T$,
$\Omega_{13}=PF$, $\Omega_{14}=PG$, $\Omega_{17}=A^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3]$,
$\Omega_{22}=-(1-h_d)Q_3+\varepsilon_2\beta^2I+\delta hX_{11}+X_{13}+X_{13}^T+\delta hZ_{22}-Z_{23}-Z_{23}^T$,
$\Omega_{25}=\delta hX_{12}-X_{13}+X_{23}^T$, $\Omega_{27}=B^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3]$,
$\Omega_{33}=-\varepsilon_1I$,
$\Omega_{37}=F^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3]$, $\Omega_{44}=-\varepsilon_2I$,
$\Omega_{47}=G^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3]$,
$\Omega_{55}=Q_2-Q_1+\delta hX_{22}-X_{23}-X_{23}^T+(1-\delta)hY_{11}+Y_{13}+Y_{13}^T$,
$\Omega_{56}=(1-\delta)hY_{12}-Y_{13}+Y_{23}^T$, $\Omega_{66}=-Q_2+(1-\delta)hY_{22}-Y_{23}-Y_{23}^T$,
$\Omega_{77}=-[\delta hR_1+(1-\delta)hR_2+\alpha hR_3]$.

Proof. In Case I, a Lyapunov–Krasovskii functional candidate can be constructed as

$$V(t)=V_1(t)+V_2(t)+V_3(t) \tag{11}$$

where

$$V_1(t)=x^T(t)Px(t)$$
$$V_2(t)=\int_{t-\delta h}^t x^T(s)Q_1x(s)ds+\int_{t-h}^{t-\delta h}x^T(s)Q_2x(s)ds+\int_{t-h(t)}^t x^T(s)Q_3x(s)ds$$
$$V_3(t)=\int_{-\delta h}^0\int_{t+\theta}^t\dot{x}^T(s)R_1\dot{x}(s)dsd\theta+\int_{-h}^{-\delta h}\int_{t+\theta}^t\dot{x}^T(s)R_2\dot{x}(s)dsd\theta+\int_{-h(t)}^0\int_{t+\theta}^t\dot{x}^T(s)R_3\dot{x}(s)dsd\theta$$

Time derivative of $V(t)$ for $t\in[0,\infty]$ along the trajectory of (1) yields

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t) \tag{12}$$

where

$$\dot{V}_1(t)=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)$$
$$=2x^T(t)P[Ax(t)+Bx(t-h(t))+Ff(x(t),t)+Gg(x(t-h(t)),t)]$$