ISA Transactions 52 (2013) 335–341

Contents lists available at SciVerse ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research Article

# Chattering free adaptive multivariable sliding mode controller for systems with matched and mismatched uncertainty

Sanjoy Mondal *, Chitralekha Mahanta

*Department of Electronics and Electrical Engineering, Indian Institute of Technology Guwahati, Guwahati 781039, India*

A R T I C L E I N F O

*Article history:*
Received 28 September 2012
Received in revised form
4 December 2012
Accepted 19 December 2012
Available online 26 January 2013
This paper was recommended for publication by Dr. Jeff Pieper

*Keywords:*
Sliding mode control
Mismatched uncertainty
Adaptive tuning
Vertical take-off and landing (VTOL) aircraft

A B S T R A C T

In this paper, a chattering free adaptive sliding mode controller (SMC) is proposed for stabilizing a class of multi-input multi-output (MIMO) systems affected by both matched and mismatched types of uncertainties. The proposed controller uses a proportional plus integral sliding surface whose gain is adaptively tuned to prevent overestimation. A vertical take-off and landing (VTOL) aircraft system is simulated to demonstrate the effectiveness of the proposed control scheme.



## 1. Introduction

Physical systems suffer from performance degradation and instability due to uncertainties existing in nature which can be broadly classified into matched and mismatched types. Uncertainties acting on the system through the input channel are called matched uncertainties, whereas perturbations in the system parameters are termed as mismatched uncertainties. Classical control techniques like adaptive control [1], optimal control [2], sliding mode control [3] and intelligent control methods like fuzzy logic control [4] have been extensively used in control systems perturbed by matched uncertainty. Among these methods, sliding mode control has received wide acceptance owing to its robustness and simplicity. However, designing sliding mode controllers for systems perturbed by the mismatched type of uncertainty still remains a challenge to the research community. The difficulty lies in the fact that the dynamics of the uncertain system are affected even after reaching the sliding mode.

Active research is continuing in the control community for developing sliding mode controllers for multi-variable systems affected by mismatched type of uncertainty [5–8]. One significant research finding is that the stability of the system is guaranteed if the system trajectory is driven to a bounded region [9–11]. Hence to ensure asymptotic stability, restriction of keeping an upper bound on uncertainties is imposed in most of the research works. By designing a sliding mode controller for certain states of the system which are provided as inputs to a reduced order system can take care of the mismatched uncertainties. However, limitation of this method is that uncertainties should lie in the range space of certain matrix of the nominal system [3]. A fuzzy logic-based sliding mode controller proposed in [11] was successful in achieving quadratic stability for systems with mismatched uncertainty. Even this method could handle mismatched uncertainty of a certain form only provided its bound was known a priori [12–14]. By introducing two sets of switching surfaces for the subsystems and hence reducing the rank of the uncertainty, asymptotic stability was achieved in [15]. Dynamic output feedback sliding mode controllers were attempted in [16] and nonlinear integral type sliding surface was used to deal with mismatched uncertainties in [17]. All these works required prior knowledge about the upper bound of the mismatched uncertainty which is in general difficult to obtain. Hence, a strategy to obtain the upper bound of the system uncertainty or a method that does not require this knowledge is needed. The adaptive sliding mode controller proposed in [18–20] provided a solution to this problem. However, this adaptive method yielded gains which were overestimated in many cases giving rise to large control efforts and high chattering [21,22].

Although the sliding mode controller guarantees robustness, chattering is its main drawback. Chattering is the high frequency bang-bang type of control action which leads to premature wear and tear or even breakdown of the system being applied to. Chattering is caused due to the fast dynamics which are usually neglected in the ideal model utilizing digital controllers with a finite sampling rate. This disadvantage of chattering could be reduced by techniques such as nonlinear gains, dynamic

* Corresponding author. Tel.: +91 361 258 2507; fax: +91 361 2582542.
*E-mail addresses:* m.sanjoy@iitg.ac.in, mondal.sanjoy@gmail.com (S. Mondal), chitra@iitg.ernet.in (C. Mahanta).

0019-0578/$ - see front matter © 2013 ISA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.isatra.2012.12.007

extensions or by using more recent strategies such as higher order sliding mode control. In [23,24] an algorithm has been proposed based on the block control and quasi-continuous higher order sliding mode techniques for nonlinear systems subjected to mismatched uncertainty. The core idea that drives the higher order sliding mode control is that it keeps the sliding surface as well as its higher order derivatives to zero. The higher order sliding mode controller ensures good tracking performance, robustness and finite time stabilization of the controlled system. Past few decades witnessed tremendous improvement in the second-order sliding mode (SOSM) controller. Twisting and super twisting [25], suboptimal [26,27], drift algorithm [28–30] are the existing SOSM control algorithms. Nonlinear sliding surface is mostly used to design a second-order sliding mode controller for uncertain systems. Wen and Cheng [19] proposed an adaptive variable structure controller for a class of dynamic systems with matched and mismatched perturbations. The controller proposed by Wen and Cheng [19] achieved asymptotic stability without having prior knowledge about the upper bounds of perturbations. However, this control scheme suffered the drawback of severe chattering in the control input. Similar kind of problem was cited in [20] too.

The major contributions of this paper are the following:

- An adaptive integral sliding mode controller (SMC) is proposed for stabilization of a class of MIMO systems affected by both matched and mismatched uncertainties.
- An adaptive tuning law is designed and by using that law the mismatched perturbations are rejected during the sliding mode while ensuring asymptotical stability of the overall system.
- The adaptive tuning law ensures that there is no gain overestimation with respect to the unknown uncertainties.
- The proposed controller eliminates chattering in the control input and hence is suitable for practical applications.

The design procedure can be divided into two steps. The first step is to build the sliding surface using an adaptive technique that eliminates the need of prior knowledge about the upper bounds of system perturbations except for those at the input. In the next step, a derivative control law is developed which contains the discontinuous sign function. The actual control is obtained by integrating the derivative control and thereby the control becomes continuous, smooth and chattering free.

The outline of this paper is as follows. Section 2 describes the system and the problem is formulated. The design procedure for the proposed adaptive integral sliding mode controller (SMC) is explained in Section 3. Effectiveness of the proposed controller is demonstrated in Section 4 by performing simulation studies. Conclusions are drawn in Section 5.

## 2. System description and problem formulation

Let us consider the following dynamic system:

$$\dot{x}(t) = Ax + B[u + \xi(t,x)] + p(t,x) \tag{1}$$

where $x \in R^n$ is the state vector and $u \in R^m$ is the control input. Moreover, $A$ and $B$ are known matrices with proper dimension and $B$ has full rank. Furthermore, $\xi(t,x)$ and $p(t,x)$ represent the unknown matched and mismatched uncertainties, respectively. The objective of the proposed control scheme is to design an adaptive chattering free sliding mode scheme for a class of MIMO systems with matched and mismatched perturbations. The design of the sliding mode controller involves two key steps, viz. (i) designing the sliding surface and (ii) designing the control input which obeys the reaching law property that the sliding manifold approaches zero in finite time.

The sliding surface $\sigma$ is designed as

$$\sigma = Sx \tag{2}$$

where $S \in R^{m*n}$ is a constant matrix designed by selecting the eigenvalues suitably (all negative) to make the system stable [31]. By using the coordinate transformation $\left[\begin{smallmatrix} z \\ \sigma \end{smallmatrix}\right] = Mx$, where the transformation matrix $M = \left[\begin{smallmatrix} W_g \\ B_g \end{smallmatrix}\right]$, Eq. (1) can be transformed to

$$\begin{aligned} \dot{z} &= W_g AWz + W_g AB\sigma + W_g p(t,x) \\ \dot{\sigma} &= B_g AWz + B_g AB\sigma + u + \xi(t,x) + Sp(t,x) \end{aligned} \tag{3}$$

here $S = B_g$ and $W_g$, $B_g$ satisfy $B_g B = I_m$, $B_g W = 0$, $W_g B = 0$, and $W_g W = I_{n-m}$. The matrix $W$ is chosen in such a way that $J = W_g AW$ has the desired eigenvalues [32], where $J$ is a symmetric matrix. It can be verified that

$$M^{-1} = [W \;\; B] \tag{4}$$

and it can be observed that $x = Wz + B\sigma$.

When the system is in the sliding mode, it satisfies the conditions $\sigma = 0$ and $\dot{\sigma} = 0$. Then, the perturbation term in Eq. (3) becomes $W_g p(t,x) = W_g p(t,Wz) = p_r(t,z)$. Now the reduced order equation becomes

$$\dot{z} = Jz + p_r(t,z) \tag{5}$$

If the mismatched perturbation $p_r(t,z)$ satisfies $\|p_r(t,z)\| \le \phi_r \|z\|$, where $\phi_r < -\lambda_{\max}(J)$, $\lambda_{max}(J)$ being the maximum eigenvalue of the $J$ matrix, then by choosing the Lyapunov function $V = (1/2)\|z\|^2$, it can be proved that [19,20]

$$\dot{V} = z^T Jz + z^T p_r(t,z) \le \lambda_{\max}(J)\|z\|^2 + \phi_r \|z\|^2 = [\lambda_{\max}(J) + \phi_r]V < 0 \tag{6}$$

The above condition means that the system will be asymptotically stable once the sliding mode is reached. However, it is obvious from the above discussion that the sliding surface design requires the bounds of the uncertainties to be known a priori [33] which is extremely difficult practically. Hence, the need arises for designing the sliding surface in such a way that prior knowledge about the bounds of the uncertainties is not required.

### 2.1. The adaptive sliding surface design

Let us consider the sliding surface

$$\sigma = S(t)x \tag{7}$$

The sliding coefficient matrix $S(t) \in R^{m*n}$ can be designed as [19]

$$S(t) = B^+ + N(t)W_g \tag{8}$$

where $B^+ = (B^T B)^{-1} B^T \in R^{m*n}$ is the Moore–Penrose pseudo-inverse [34] of $B$ and $N(t) \in R^{m*n}$ is designed using an adaptive technique to be explained later. Let us consider the transformation

$$\begin{bmatrix} z \\ \sigma \end{bmatrix} = \begin{bmatrix} W_g \\ S(t) \end{bmatrix} x = M(t)x \tag{9}$$

Now defining $W(t) = W_g^+ - BN(t) \in R^{n*(n-m)}$ and $W_g^+ = W_g^T (W_g W_g^T)^{-1} \in R^{n*(n-m)}$, it can be verified that

$$M(t)^{-1} = [W(t) \;\; B] \tag{10}$$

From (9) and (10), it can be observed that

$$x = W(t)z + B\sigma \tag{11}$$

So, Eq. (1) gets transformed to

$$\dot{z} = W_g AW(t)z + W_g AB\sigma + W_g p(t,x) \tag{12}$$

$$\dot{\sigma} = S(t)AW(t)z + S(t)AB\sigma + u + \dot{N}(t)z + \xi(t,x) + S(t)p(t,x) \tag{13}$$

When the system is in the sliding mode, it satisfies the conditions $\sigma = 0$ and $\dot{\sigma} = 0$. Then, the perturbation term in Eq. (12) becomes $W_g p(t,x) = W_g p(t,W(t)z) = \overline{p}(t,z)$ and Eq. (12) transforms into a