ISA Transactions 52 (2013) 375–382

Contents lists available at SciVerse ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research Article

Design of a robust observer-based modified repetitive-control system

Lan Zhou [a,b,*], Jinhua She [c], Min Wu [a], Yong He [a]

[a] School of Information Science and Engineering, Central South University, Changsha 410083, China
[b] School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan 411201, China
[c] School of Computer Science, Tokyo University of Technology, Tokyo 192-0982, Japan

ARTICLE INFO

Article history:
Received 16 July 2011
Received in revised form
6 January 2013
Accepted 14 January 2013
Available online 11 February 2013
This paper was recommended for publication by Dr. Q.-G. Wang

Keywords:
Repetitive control
State-observer
Two-dimensional system
Linear matrix inequality (LMI)
Robust stability

ABSTRACT

This paper concerns the design of a robust observer-based modified repetitive-control system for a class of strictly proper plants with periodic time-varying uncertainties. It exploits the inherent structure characteristics of repetitive control to convert the design problem to a robust stabilization problem for a continuous-discrete two-dimensional system. The singular-value decomposition and Lyapunov stability theory are used to derive a linear-matrix-inequality-based design algorithm for the parameters of control and state-observer gains. Two tuning parameters are introduced to perform the preferential adjustment of control and learning. A numerical example illustrates the adjusting procedure and demonstrates the validity of the method.

1. Introduction

Repetitive control (RC) is a method of tracking control for systems that exhibit repetitive behavior, such as servo systems. In a repetitive-control system (RCS, Fig. 1), repetitive controller contains a pure-delay positive-feedback loop, which is an internal model of a periodic signal with its harmonics at angular frequencies of $\omega_k = 2k\pi/T, k = 0, 1, \ldots$. For the periodic signal, $C_R(j\omega_k) = \infty$. This guarantees the perfect tracking in the steady state.

From the viewpoint of control theory, an RCS is a neutral-type delay system. It can be stabilized only when the relative degree of a plant is zero. To stabilize a strictly proper plant, which is the case in most control engineering applications, a low-pass filter has to be inserted in the delay line. This yields a modified repetitive controller. The resulting system is called a modified RCS (MRCS) [1]. Since a modified repetitive controller is just an approximate model of a periodic signal, perfect tracking is expected and there exists a steady-state tracking error, that is, in an MRCS, the low-pass filter relaxes the stabilization condition, but degrades the tracking precision [2]. So, it has theoretical and practical significance to develop a method of designing an MRCS that ensures the robust stability and improves the tracking performance of the system.

RC actually involves two types of actions: continuous control within one repetition period and discrete learning between two periods. Wu et al. [3,4] described control and learning in two-dimensional (2D) space and constructed a continuous-discrete 2D model that converted the design of a robust RCS to a robust stabilization problem of a continuous-discrete 2D system. The adjustment of control and learning improved the system convergence and tracking performance. But they just considered the case in which the plant has a relative degree of zero. This restricts the applications of the result because many plants have a relative degree larger than zero. Since we need to insert a low-pass filter in the delay line in an MRCS to guarantee the stability, and the low-pass filter mixes control and learning together, it makes it impossible to describe control and learning separately. Thus, the method in [3,4] cannot be directly extended to the design of an MRCS for a strictly proper plant.

For a nominal MRCS, [5] presented an iterative algorithm to calculate the maximum cutoff angular frequency of the low-pass filter and the corresponding stabilizing controller gains. However, the whole state of the plant is needed for the design of the controllers, which unfortunately is unavailable in many practical applications. In addition, uncertainties exist widely in control practice, such as parameter perturbation, modeling error, etc.

To widen the practical range of RC, it is important to solve the problem of designing a robust RC law for an MRCS by employing the output signal of the plant.

This paper presents a method of designing a robust observer-based MRCS for a class of linear plants with time-varying periodic

* Corresponding author at: School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan 411201, China.
Tel.: +86 731 58290114; fax: +86 731 58290509.
E-mail addresses: zlly98@yahoo.cn (L. Zhou), she@cs.teu.ac.jp (J. She), min@csu.edu.cn (M. Wu), heyong@csu.edu.cn (Y. He).

0019-0578/$ - see front matter © 2013 ISA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.isatra.2013.01.002

uncertainties. First, employing continuous lifting method, we construct a continuous-discrete 2D model to describe the structural characteristics of the observer-based MRCS and obtain a 2D control law that allows preferential adjustment of control and learning by means of control gains. Next, based on the 2D model, combining the continuity of RC and the singular-value decomposition (SVD) of the output matrix, we derive a sufficient condition for robust stability in the form of a linear matrix inequality (LMI). Two tuning parameters contained in the LMI enable the simple and intuitive adjustment of control and learning. A numerical example is used to show the adjustment effectiveness and the consideration of the parameters selection. The sensitivity function of the MRCS for the nominal plant verifies the parameter adjustment in the frequency domain.

2. Problem description

Consider the MRCS in Fig. 2. $r(t)$ is a given periodic reference signal with a period of T:

$$e(t) = r(t) - y(t)$$

is the tracking error. The modified repetitive controller is

$$C_{MR}(s) = \frac{1}{1 - q(s)e^{-sT}}. \tag{1}$$

$q(s)$ is a first-order low-pass filter

$$q(s) = \frac{\omega_c}{s + \omega_c}, \tag{2}$$

where ω_c is the cutoff angular frequency of the filter. Thus, the state-space representation of $C_{MR}(s)$ is

$$\begin{cases} \dot{x}_f(t) = -\omega_c x_f(t) + \omega_c x_f(t-T) + \omega_c e(t), \\ v(t) = e(t) + x_f(t-T), \end{cases} \tag{3}$$

where $x_f(t)$ is the state variable of the low-pass filter, and $v(t)$ is the output of the modified repetitive controller.

In Fig. 2, the compensated single-input, single-output (SISO) plant with time-varying structured uncertainties is

$$\begin{cases} \dot{x}_p(t) = [A + \Delta A(t)]x_p(t) + [B + \Delta B(t)]u(t), \\ y(t) = Cx_p(t), \end{cases} \tag{4}$$

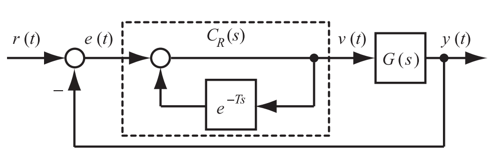

Fig. 1. Configuration of basic repetitive-control system.

where $x_p(t) \in \mathbb{R}^n$ is the state variable; $u(t), y(t) \in \mathbb{R}$ are the control input and output variables, respectively. A, B, and C are real constant matrices. We assume that (A,B) is controllable and (C,A) is observable, which are standard for a servo system. Assume that the uncertainties of the plant are

$$[\Delta A(t) \;\; \Delta B(t)] = MF(t)[N_0 \;\; N_1], \tag{5}$$

where M, N_0, and N_1 are known constant matrices; and $F(t) \in \mathbb{R}^{n \times n}$ is a real, unknown, and possibly time-varying matrix with Lebesgue measurable elements:

$$F^T(t)F(t) \le I, \quad \forall t > 0. \tag{6}$$

We make the following assumption.

Assumption 1. The uncertainties, $\Delta A(t)$ and $\Delta B(t)$, vary periodically with the repetition period of the MRCS in Fig. 2; that is,

$$\Delta A(t+T) = \Delta A(t), \quad \Delta B(t+T) = \Delta B(t), \quad \forall t > 0. \tag{7}$$

Assumption 1 holds in many applications, for example, a pulse width modulation (PWM) inverter subjects to a periodic load disturbance [6], and the parameters of a chuck-workpiece system change periodically when the stiffness of the system varies periodically during the cutting process [7,8], etc.

The following state observer is used to reproduce the state of the plant

$$\begin{cases} \dot{\tilde{x}}_p(t) = A\tilde{x}_p(t) + Bu(t) + L[y(t) - \tilde{y}(t)], \\ \tilde{y}(t) = C\tilde{x}_p(t), \end{cases} \tag{8}$$

where L is the observer gain.

The error between the states of the actual plant and the observer is

$$x_\delta(t) = x_p(t) - \tilde{x}_p(t).$$

State equations (4) and (8) yield

$$\dot{x}_\delta(t) = \Delta A(t)\tilde{x}_p(t) + [A + \Delta A(t) - LC]x_\delta(t) + \Delta B(t)u(t). \tag{9}$$

A linear control law based on the state of the observer and the output of the modified repetitive controller is

$$u(t) = K_e v(t) + K_p \tilde{x}_p(t), \quad K_e \in \mathbb{R}, \; K_p \in \mathbb{R}^{1 \times n}, \tag{10}$$

where K_e is the feedback gain of the modified repetitive controller and K_p is the reconstructed-state feedback gain.

Remark 1. Due to the existence of periodic uncertainties, Plant (4) generates undesired high harmonics, which are integral multiple of the fundamental frequency, i.e., $\omega_k = 2k\pi/T$, $k = 1,2,3,\ldots$. So, the harmonic components has to be added to the control input $u(t)$ in (10) to cancel the harmonic effects even if the

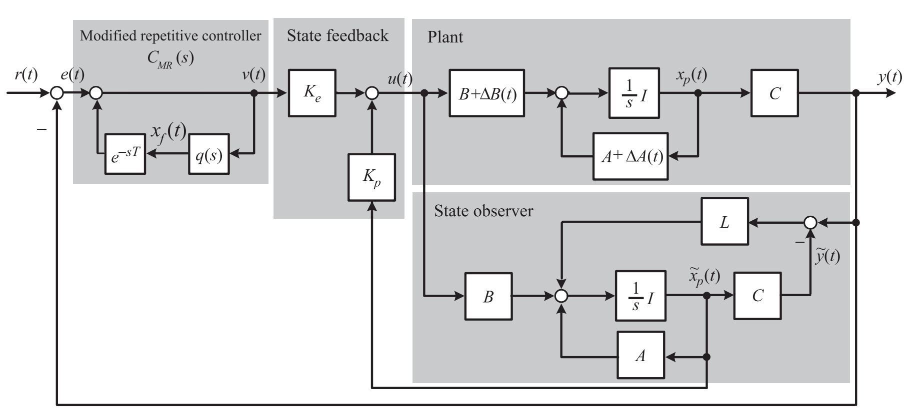

Fig. 2. Configuration of state-observer-based modified repetitive-control system.