



## Practice Article

# Time-scale separation of a class of robust PD-type tracking controllers for robot manipulators



Salvador González-Vázquez, Javier Moreno-Valenzuela \*

*Instituto Politécnico Nacional-CITEDI, Av. del Parque 1310, Mesa de Otay, Tijuana, BC, Mexico*

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## ABSTRACT

In this paper, the trajectory tracking control of robot manipulators is studied from the theoretical and practical point of view. By using the theory of singularly perturbed systems, a class of PD-type robust controllers is introduced. Our analysis departs from parameterizing the proportional and derivative gains with a perturbing parameter. We prove that the smaller the value of perturbing parameter, the smaller the ultimate bound of the joint position tracking error. Derived from the introduced analysis, two forms of extending the proposed class of controllers are discussed. In one, error-varying PD gains are considered while in the another one, a dynamic extension to avoid joint velocity measurements is incorporated. An experimental study in a planar two degrees-of-freedom direct-drive robot is also presented. Under similar implementation conditions, four controllers are tested. The best performance is obtained for a nonlinear PD controller derived from the proposed class of controllers.

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## 1. Introduction

Robot tracking control consists in the robot joint position tending asymptotically to a time-varying desired position as time increases. However, asymptotic tracking cannot be assured with a smooth model-based control law if the robot model parameters are partially known. Hence, in first instance, the process of implementing a model-based motion controller is achieved by using a complex experimental phase where estimation of the robot model parameters is carried out.

Implementations of an identification algorithm and a model-based controller may be time-consuming goals; particularly if the number of degrees-of-freedom is high. So, in the last 20 years there has been a mathematical and practical interest in studying simple control architectures, linear and nonlinear ones, with the aim of achieving robot tasks with relative tolerance to inaccuracies in the executed trajectories. Typical examples of such a controllers are the linear proportional–derivative (PD) and proportional–integral–derivative (PID) on the position error [1,2].

An inspection of the literature shows that linear and nonlinear approaches with the common feature of maintaining a PD-type structure on the position error have been devised to achieve

robust control of robot manipulators. A review of some key results on PD-type control is provided next. The linear PD control used to follow a time-varying desired position was studied in the paper by Qu and Dorsey [3], where special parameterizations of the gains were proposed. Their main result was to prove that the controller can make the tracking error uniformly ultimately bounded. In order to reduce the ultimate bound of the tracking error, controllers with feedforward compensation plus linear PD control structured have been proposed. For example, in [4], the adaptive computed feedforward plus linear PD control was studied. On the other hand, concerning nonlinear PD-type controllers, the papers [5,6] have introduced generalizations. The solution proposed there relies in controllers expressed in terms of generalized PD nonlinear functions should satisfy a Lyapunov-based closed-loop stability analysis. Besides, nonlinear PD controllers based on varying gains have been proposed. For example, Qu et al. [7] introduced a controller with the gains growing quadratically with respect to the position and velocity tracking errors. However, this algorithm has a saturated time-varying term, which may approach a sign function and, in consequence, introduce high frequency components to the control action. Later, the need of using a saturated time-varying term was removed by Wang et al. [8], by introducing a PD controller based on an auxiliary-polynomial gain design. More recently, a hybrid fuzzy nonlinear PD controller was developed in [9], by combining two nonlinear tracking differentiators to a conventional fuzzy PD controller. A comparative study of PD-based controllers (linear

\* Corresponding author. Tel.: +52 664 6231344x82806; fax: +52 664 6231388.  
E-mail addresses: [salvador\\_g\\_v@hotmail.com](mailto:salvador_g_v@hotmail.com) (S. González-Vázquez),  
[moreno@citedi.mx](mailto:moreno@citedi.mx) (J. Moreno-Valenzuela).

and nonlinear ones) was conducted in the work by Ouyang and Zhang [10], where a PD-type plus learning compensation presented the best performance. The papers [11,12] proposed nonlinear motion controllers with a PD-type control action designed to mitigate the possible effects of actuator saturation. The robust trajectory tracking control of electrical manipulators was addressed [13], where a high-gain control scheme is proposed. In [14] a fuzzy logic control law is proposed to achieve trajectory tracking of a class of electromechanical mechanical systems. In [15], under a complex analysis based on error dynamics, a decentralized robust tracking controller consisting of a nominal controller and a robust compensator is proposed. An adaptive control algorithm is developed in [16] using a complex parameter estimation rule. The combination of fuzzy control and an integral sliding mode control algorithm is explored with good results in [17]. In [18], a variable structure redesign of the robust controller discussed in [19] is introduced. Robust discontinuous PD-type scheme have recently been studied in the work [20]. Other recent approaches based on PD control are found in [21–30].

On the other hand, notice that the practical implementation of variable structure controllers is achieved by smoothing the sign function and the controller discontinuities [28], which produces local high-gain controllers. Industrial practice also suggests the use of high-gain controllers, since many processes are subjected nonlinearities. Thus, high-control is form the linearize the closed-loop system.

As alternative to Lyapunov's theory, the theory of singularly perturbed systems has been recognized as a powerful in the analysis and design of robot controllers. Essentially, this technique is based in analyzing the convergence of the solution of differential equation in two time-scales [31].

Usually, the performance and viability of control algorithms are usually validated by means of numerical simulations. However, the disadvantage of using numerical simulations is that many effects that appear in practice are neglected. Hence the performance of a control scheme is completely different from one obtained by simulations. Thus, the inclusion of real-time experiments in the proposal of new theory for robot control is important, in particular if comparisons with respect to standard control approaches, such as the linear PID control, are also presented. These observations have motivated us to complement our theoretical results with an experimental study.

The study presented in this paper concerns control architectures that are continuous-time, smooth and structured with PD terms in the trajectory tracking error. In fact, we show that controllers structured with a feedforward plus PD terms exhibit a time-scale separation. Known controllers and new ones are addressed through the proposed methodology. More specifically, the contributions of this paper are as follows:

1. Derived from the perspective of the theory of singularly perturbed systems, a class of PD-type robust controllers is proposed.
2. By using real-time experimental evaluations, an assessment of the proposed class of controllers is provided. Experiments were carried out in a planar horizontal two degrees of freedom direct-drive robot is presented. We compare linear robust controllers (including the PID) with respect to nonlinear designs.

The proposed class of controllers includes known controllers and allows designing new ones. Specifically, we prove that the controllers proposed in [3,8] are in the proposed control frameworks, and therefore, exhibit such a time-scale separation. In addition, the proposed class of controllers is extended in two directions. In the first one, error-varying PD gains are incorporated, while in the second one, the case of partial-state feedback is solved.

Let us notice that the idea that PD-type controllers can be analyzed by using the singular perturbation approach has been used in [11,12,32–35]. However, the formulation of the conditions for a generalization of controllers based on the singular perturbation approach has not received too much attention.

On the other hand, concerning experimental work, we show that under similar implementation conditions a nonlinear PD-type controller exhibits better performance than a linear PID algorithm.

This paper is organized as follows. Section 2 is devoted to the robot model and control objective. Section 3 concerns the mathematical preliminaries. In Section 4, the proposed class of controllers is discussed. Extensions of the PD controllers are given in Section 5. The experimental evaluation in a robot manipulator is described in Section 6, while some concluding discussions are drawn in Section 7.

Throughout this paper the following notation will be adopted.  $\|\mathbf{x}\| = \sqrt{\mathbf{x}^T \mathbf{x}}$  stands for the Euclidean norm of vector  $\mathbf{x} \in \mathbb{R}^n$ . The notation  $B_r$  denotes the set given by the ball  $\{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}\| \leq r\}$ . Order of magnitude  $O(\epsilon)$  means that the inequality

$$\|\mathbf{x}(t, \epsilon) - \mathbf{x}_0(t)\| \leq k\epsilon, \quad \mathbf{x} \in \mathbb{R}^n$$

is satisfied for all  $0 < \epsilon < \epsilon^*$  and some  $k > 0$ . The vector  $\mathbf{x}(t, \epsilon)$  is the solution of the system

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \epsilon), \quad \mathbf{x}(t_0) = \gamma(\epsilon),$$

$\mathbf{x}_0(t)$  is the solution of the system with  $\epsilon = 0$ , i.e.,

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, 0), \mathbf{x}(t_0) = \gamma(0),$$

and the constant  $\epsilon^*$  guarantees the uniqueness of the solution  $\mathbf{x}(t, \epsilon)$  for all  $t \geq 0$  and  $0 < \epsilon < \epsilon^*$ .

## 2. Robot dynamics and control goal

### 2.1. Robot dynamics

The dynamics in joint space of a serial-chain  $n$ -link robot manipulator can be written as [19,36],

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + F_v \dot{\mathbf{q}} = \boldsymbol{\tau}, \quad (1)$$

where  $\mathbf{q}$  is the  $n \times 1$  vector of joint displacements,  $\dot{\mathbf{q}}$  is the  $n \times 1$  vector of joint velocities,  $M(\mathbf{q})$  is the  $n \times n$  symmetric positive definite manipulator inertia matrix,  $C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$  is the  $n \times 1$  vector of centripetal and Coriolis torques,  $\mathbf{g}(\mathbf{q})$  is the  $n \times 1$  vector of gravitational torques,  $F_v$  is a  $n \times n$  diagonal positive definite matrix which contains the viscous friction coefficients of each joint and  $\boldsymbol{\tau}$  is the  $n \times 1$  vector of applied control inputs. Without loss of generality, let us assume that the robot is actuated by permanent-magnet direct-current (DC) motors. Also, assume that the motors are operated by a servo amplifier in current mode that the actual current is equal to the desired one. Therefore, the DC motor output torque is given as

$$\boldsymbol{\tau}(t) = K\mathbf{u}(t), \quad (2)$$

with  $K$  a  $n \times n$  diagonal positive definite matrix which is proportional to the motor constants and to the servo amplifier gains and  $\mathbf{u} \in \mathbb{R}^n$  is the vector of input voltages to the servo amplifiers. See the reference [37] for a discussion on the obtention of Eq. (2).

### 2.2. Control goal formulation

The following definition will be important [31]: The solutions of the system  $\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x})$  are said to be uniformly ultimately bounded if there exist positive constants  $b$  and  $c$ , and for every  $\alpha \in (0, c)$  there is a positive constant  $T = T(\alpha, b)$  such that

$$\|\mathbf{x}(t_0)\| < \alpha \Rightarrow \|\mathbf{x}(t)\| \leq b \quad \forall t \geq t_0 + T. \quad (3)$$

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