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Improved delay-dependent robust stability criteria for recurrent neural networks with time-varying delays

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ABSTRACT

In this paper, the problem of improved delay-dependent robust stability criteria for recurrent neural networks (RNNs) with time-varying delays is investigated. Combining the Lyapunov–Krasovskii functional with linear matrix inequality (LMI) techniques and integral inequality approach (IIA), delay-dependent robust stability conditions for RNNs with time-varying delay, expressed in terms of quadratic forms of state and LMI, are derived. The proposed methods contain the least numbers of computed variables while maintaining the effectiveness of the stability conditions. Both theoretical and numerical comparisons have been provided to show the effectiveness and efficiency of the present method. Numerical examples are included to show that the proposed method is effective and can provide less conservative results.

1. Introduction

In recent years, neural networks (NNs) have attracted much attention in research and have found successful applications in many areas such as pattern recognition, image processing, association, optimization problems [6,16]. One of the important research topics is the globally asymptotic stability of the neural network models. However, in the implementation of artificial NNs, time delays are unavoidable due to the finite switching speed of amplifiers. It has been shown that the existence of time delays in recurrent neural networks (RNNs) may lead to oscillation, divergence or instability. Therefore, the stability of RNNs with delay has become a topic of great theoretical and practical importance. Generally, when a neural network is applied to solve an optimization problem, it needs to have a unique and globally stable equilibrium point. Thus, it is of great interest to establish conditions that ensure the global asymptotic stability of a unique equilibrium point of RNNs with delay [1,2,4,5,7–15,18–23].

So far, the stability criteria of RNNs with time delay are classified into two categories, i.e., delay independent [1,2,4,14,18,23] and delay dependent [5,7,8,10–13,15,20,21]. Generally speaking, the delay-dependent stability criteria are less conservative than delay-independent when the time-delay is small. Therefore, authors always consider the delay-dependent type. Some less conservative stability criteria were proposed in [8] by considering some useful terms and using the free-weighting matrices method. The stability criteria for neural networks with time-varying delay were considered in [10] where the relationship between the time-varying delay and its lower and upper bound was taken into account. By constructing a new augmented Lyapunov functional which contains a triple-integral term, an improved delay-dependent stability criterion is derived in [19]. However, these results have conservatism to some extent, which exist room for further improvement.

In this paper, the problem of delay-dependent robust stability criterion for recurrent neural networks with time-varying delay is considered. A sufficient condition for the solvability of this problem, which depends on the size of the time delay, has been presented by means of the Lyapunov functional and the linear matrix inequality (LMI) approach. Furthermore, the proposed condition in this paper is less conservative than previously established ones and include the least number of variables, which has been shown by some numerical examples. All results are derived in the LMI framework and the solutions are obtained by using LMI toolbox of Matlab. Finally, numerical examples are given to indicate significant improvements over the existing results.

2. Problem formulation

Consider the following recurrent neural network with time-varying delays and parameter uncertainties:

$$\begin{aligned}\dot{u}(t)=&-(C+\Delta C(t))u(t)+(A+\Delta A(t))f(u(t))\\&+(B+\Delta B(t))f(u(t-h(t)))+J,\end{aligned}\tag{1}$$

where $u(t)=[u_1(t),\dots,u_n(t)]^T\in R^n$ is the state vector with the n neurons; $f(u(t))=[f_1(u_1(t)),\dots,f_n(u_n(t))]^T\in R^n$ is called an activation

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function indicating how the jth neuron responses to its input; $C=\mathrm{diag}(c_1,\ldots,c_n)$ is a diagonal matrix with each $c_i>0$ controlling the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $A=(a_{ij})_{n\times n}$, and $B=(b_{ij})_{n\times n}$ are the feedback and the delayed feedback matrix, respectively; $J=[J_1,\ldots,J_n]^T\in R^n$ is a constant input vector, $\Delta A(t)$, $\Delta B(t)$, and $\Delta C(t)$ are unknown matrices that represent the time-varying parameter uncertainties and $h(t)$ is the time delay of the system satisfies

$$0\le h(t)\le h,\quad \dot h(t)\le h_d, \tag{2}$$

where h and h_d are some positive constants.

In this paper, the neuron activation functions are assumed to be bounded and satisfy the following assumption.

Assumption 1. It is assumed that each of the activation functions $f_j (j=1,2,\ldots,n)$ possess the following condition

$$0\le \frac{f_i(\varsigma_1)-f_i(\varsigma_2)}{\varsigma_1-\varsigma_2}\le k_i,\ \varsigma_1\ne\varsigma_2\in R, i=1,2,\ldots,n, \tag{3}$$

where $k_i (i=1,2,\ldots,n)$ are known constant scalars.

Next, the equilibrium point $u^*=[u_1^*,\ldots,u_n^*]^T$ of system (1) is shifted to the origin through the transformation $x(t)=u(t)-u^*$, then system (1) can be equivalently written as the following system

$$\dot x(t)=-(C+\Delta C(t))x(t)+(A+\Delta A(t))g(x(t))+(B+\Delta B(t))g(x(t-h(t))), \tag{4}$$

where $x(\cdot)=[x_1(\cdot),\ldots,x_n(\cdot)]^T$, $g(x(\cdot))=[g_1(x_1(\cdot)),\ldots,g_n(x_n(\cdot))]^T$, $g_i(x_i(\cdot))=f_i(x_i(\cdot)+u_i^*)-f_i(u_i^*)$, $i=1,2,\ldots,n$. It is obvious that the function $g_j(\cdot)(j=1,2\ldots,n)$ satisfies the following condition,

$$0\le\frac{g_i(x_i)}{x_i}\le k_i,\quad g_i(0)=0,\quad \forall x_i\ne0,\quad i=1,2,\ldots,n, \tag{5}$$

which is equivalent to

$$g_i(x_i)(g_i(x_i)-k_ix_i)\le0,\quad g_i(0)=0,\quad \forall x_i\ne0,\quad i=1,2,\ldots,n. \tag{6}$$

The matrices $\Delta C(t)$, $\Delta A(t)$ and $\Delta B(t)$ are the uncertainties of the system and have the form

$$[\Delta C(t)\quad \Delta A(t)\quad \Delta B(t)]=DF(t)[E_c\quad E_a\quad E_b], \tag{7}$$

where D, E_c, E_a, and E_b are known constant real matrices with appropriate dimensions and $F(t)$ is an unknown matrix function with Lebesgue-measurable elements bounded by

$$F^T(t)F(t)\le I,\quad \forall t, \tag{8}$$

where I is an appropriately dimensioned identity matrix.

The following lemmas are useful in deriving the criteria. First, we introduce the following integral inequality approach (IIA), which be used in the proof of ours.

Lemma 1. [17] For any positive semi-definite matrices

$$X=\begin{bmatrix}X_{11}&X_{12}&X_{13}\\X_{12}^T&X_{22}&X_{23}\\X_{13}^T&X_{23}^T&X_{33}\end{bmatrix}\ge0, \tag{9a}$$

the following integral inequality holds

$$-\int_{t-h(t)}^{t}\dot x^T(s)X_{33}\dot x(s)ds\le \int_{t-h(t)}^{t}\begin{bmatrix}x^T(t)&x^T(t-h(t))&\dot x^T(s)\end{bmatrix}\begin{bmatrix}X_{11}&X_{12}&X_{13}\\X_{12}^T&X_{22}&X_{23}\\X_{13}^T&X_{23}^T&0\end{bmatrix}\begin{bmatrix}x(t)\\x(t-h(t))\\\dot x(s)\end{bmatrix}ds. \tag{9b}$$

Secondary, the following Schur complement result, which is essential in the proofs of Theorem 1, is introduced.

Lemma 2. [3]. The following matrix inequality

$$\begin{bmatrix}Q(x)&S(x)\\S^T(x)&R(x)\end{bmatrix}<0, \tag{10a}$$

where $Q(x)=Q^T(x)$, $R(x)=R^T(x)$ and $S(x)$ depend affine on x, is equivalent to

$$R(x)<0, \tag{10b}$$

$$Q(x)<0, \tag{10c}$$

and

$$Q(x)-S(x)R^{-1}(x)S^T(x)<0. \tag{10d}$$

Finally, the following Lemma 3 will be used to handle the parametrical perturbation.

Lemma 3. [3]. Given symmetric matrices Ω and D,E, of appropriate dimensions,

$$\Omega+DF(t)E+E^TF^T(t)D^T<0, \tag{11a}$$

for all $F(t)$ satisfying $F^T(t)F(t)\le I$, if and only if there exists some $\varepsilon>0$ such that

$$\Omega+\varepsilon DD^T+\varepsilon^{-1}E^TE<0, \tag{11b}$$

3. Main results

In this section, we use the integral inequality approach (IIA) to obtain stability criterion for a recurrent neural network with time-varying delays. First, we take up the case where $\Delta C(t)=0$, $\Delta A(t)=0$ and $\Delta B(t)=0$ in system (4) as follows:

$$\dot x(t)=-Cx(t)+Ag(x(t))+Bg(x(t-h(t))), \tag{12a}$$

$$x(t)=\phi(t),\quad t\in[-h,0]. \tag{12b}$$

Based on the Lyapunov–Krasovskii stability theorem and integral inequality approach (IIA), the following result is obtained.

Theorem 1. For given positive scalars h and h_d, the recurrent neural network system with time-varying delay (12) is asymptotically stable if there exist symmetry positive-definite matrices $P=P^T>0$, $Q=Q^T>0$, $R=R^T>0$, $Z=Z^T>0$, $U=U^T>0$, diagonal matrices $S\ge0,\Lambda_1\ge0,\Lambda_2\ge0$, and $X=\begin{bmatrix}X_{11}&X_{12}&X_{13}\\X_{12}^T&X_{22}&X_{23}\\X_{13}^T&X_{23}^T&X_{33}\end{bmatrix}\ge0$, $Y=\begin{bmatrix}Y_{11}&Y_{12}&Y_{13}\\Y_{12}^T&Y_{22}&Y_{23}\\Y_{13}^T&Y_{23}^T&Y_{33}\end{bmatrix}\ge0$, such that the following LMIs hold for

$$\Omega=\begin{bmatrix}\Omega_{11}&\Omega_{12}&\Omega_{13}&\Omega_{14}&0&\Omega_{16}\\\Omega_{12}^T&\Omega_{22}&\Omega_{23}&0&\Omega_{25}&\Omega_{26}\\\Omega_{13}^T&\Omega_{23}^T&\Omega_{33}&0&0&\Omega_{36}\\\Omega_{14}^T&0&0&\Omega_{44}&\Omega_{45}&0\\0&\Omega_{25}^T&0&\Omega_{45}^T&\Omega_{55}&0\\\Omega_{16}^T&\Omega_{26}^T&\Omega_{36}^T&0&0&\Omega_{66}\end{bmatrix}<0, \tag{13a}$$

and

$$Z-X_{33}\ge0, \tag{13b}$$

$$Z-Y_{33}\ge0, \tag{13c}$$

where

$K=\mathrm{diag}\{k_1,k_2,\ldots,k_n,\}$, $\Omega_{11}=-C^TP+PC+Q+R+hX_{11}+X_{13}+X_{13}^T$,

$\Omega_{12}=PA-C^TS+K\Lambda_1$, $\Omega_{13}=PB+K\Lambda_2$, $\Omega_{14}=hX_{12}-X_{13}+X_{23}^T$,

$\Omega_{16}=-hC^TZ$, $\Omega_{22}=U+A^TS+SA-2\Lambda_1$, $\Omega_{23}=SB$, $\Omega_{26}=hA^TZ$,