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Robust controller design for fuzzy parametric uncertain systems: An optimal control approach

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ABSTRACT

A new approach of designing a robust controller for fuzzy parametric uncertain systems is proposed. A linear time invariant (LTI) system with fuzzy coefficients is called as fuzzy parametric uncertain system (FPUS). The proposed method envisages conversion of the FPUS into an uncertain (interval) state space controllable canonical form system in terms of its alpha cut. Further, the problem of designing a robust controller is translated into an optimal control problem minimizing a cost function. For matched uncertainty, it is shown that the optimal control problem is a linear quadratic regulator (LQR) problem, which can be solved to obtain a robust controller for FPUS. The numerical examples and simulation results show the effectiveness of the proposed method in terms of robustness of the controller.

1. Introduction

An important task of feedback design is to provide robustness of closed loop control systems when a plant includes parametric uncertainties. The controlled plant represented with its mathematical model is bound to have a certain amount of uncertainty. The model is known approximately and hence the need to incorporate the robustness in design is necessary. Robustness is the property of dynamic systems to tolerate variations or uncertainties of parts of the system without exceeding predetermined tolerance bounds in the vicinity of some nominal dynamic behavior [1]. The problem of designing a robust controller for plants having unknown but bounded parameter uncertainties, which are often called the interval plants [2–5], has attracted considerable attention of researchers. Some of the reasons for uncertainties can be poor process knowledge, nonlinearities, unknown external or internal noise, environment influence, time varying parameters, changing operating conditions, measurement errors, etc. Many times, an estimation of the value of an uncertain parameter is given by an expert, such as ‘change in angle is small (or large)’. Here, the uncertain parameter can be represented by a fuzzy number. For each fuzzy number, a membership function is defined to assign a value from [0, 1] to every element in the input universe of discourse. The fuzzy representation of uncertainties indicates the interval of variations and the possibility of each different value in the variation interval by the membership function. Thus a membership value ‘1’ indicates the precise knowledge (kernel), whereas membership value ‘0’ reflects the maximum uncertainty (support) [6–8]. Such a fuzzy dynamic systems can be viewed as an extension of uncertain parametric systems or interval systems, when uncertain parameters are represented by fuzzy numbers or fuzzy coefficients and called as fuzzy parametric uncertain systems. The interval approach of representing system has the drawback of considering the range of all possible outcomes. On the other hand the approach of representing uncertainty by a fuzzy number with a membership function, also called possibilistic approach based on fuzzy set theory, is a compromise between the interval and the probabilistic approaches [9]. The membership value can be interpreted as a kind of confidence degree. Thus a value equal to 1 indicates certain knowledge whereas a value equal to 0 represents a maximum uncertainty.

In the recent literature, many approaches for analysis and design of robust controller for fuzzy parametric uncertain systems (FPUS) have been proposed [6–8,10,11]. The problem of determining the stability margin of a linear system with fuzzy parametric uncertainty has been addressed in [12,6]. Using modal interval arithmetics, a method is proposed in [7] for designing a controller under fuzzy pole placement specifications. The parametric representation of fuzzy number has been dealt in [8].

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In [10,11], a robust control for fuzzy parametric uncertain system is proposed using Kharitonov theorem. Various methods for robust control design for parametric uncertain system have been discussed in [13]. The determination of stability margin for fuzzy parametric uncertain system is also discussed in [14–17]. In [18] an inverse controller is designed for fuzzy interval systems under satisfaction of certain conditions. The controllers designed in [19] uses a nearest interval approximation method for approximating a fuzzy number. The gain, phase margin and stability analysis of FPUS based on Kharitonov's theorem are presented in [20]. Using a necessary condition and a sufficient condition developed in [2,3] for interval polynomials a robust PI/PID control for FPUS is presented in [21]. The problem of PI and PD controller design for fuzzy gain and phase margin specifications is addressed in [22]. An approach to simulate discrete linear time invariant dynamic systems with parameters and initial conditions described by fuzzy numbers is proposed in [23]. These methods of designing robust controller for fuzzy parametric uncertain systems are not optimal. The fundamental disadvantage of the method in [7] is that it cannot be used for the systems with finite zeros because in that case the corresponding system of interval linear equations cannot be solved step by step by direct substitution. In [24], a helicopter like twin rotor multi-input multi-output system is decoupled and a Takagi–Sugeno fuzzy model is obtained and then a parallel distributed fuzzy LQR is obtained. The design of a fuzzy logic controller based optimal quadratic regulator (FC-LQR) for the control of a robotic system is proposed in [25] to combine the best features of fuzzy control and LQR.

In this paper, the problem of designing a robust controller for fuzzy parametric uncertain system is converted into an optimal control approach. The uncertainties described by linguistic information are represented as fuzzy sets. The linguistic information which is approximated as a fuzzy set is represented by a unique crisp set using an interval by taking its α-cut. Thus the system with fuzzy uncertainties becomes a system with interval uncertainties for each α cut. For $\alpha=0$, we get maximum uncertainty (i.e. support) for which robust controller is designed using optimal control approach. It is shown that the designed controller is robust under this worst case condition. Obviously for any $\alpha\in[0\ 1]$, this controller will exhibit stable performance. Hence all the systems corresponding to each $\alpha\in[0\ 1]$ will be stabilized. Thus ensuring the robustness of the proposed controller for fuzzy parametric uncertain system. The suggested method is simple, involves less computational complexity and a robust controller can be designed easily.

The present paper is organized as follows. In Section 2, how a fuzzy parametric uncertainty can be represented by using its α-cuts is discussed. In Section 3, an optimal control approach used for the design of a robust controller is explained. In Section 4, numerical examples along with simulations are given to illustrate the design of a robust controller for fuzzy parametric uncertain system using proposed technique. The stability of the designed controllers is also checked using Kharitonov polynomials. Finally Section 5, concludes the paper.

2. α-cut representation of fuzzy parametric uncertainty

The system in which coefficients depend on parameters described by fuzzy function is called extended systems or fuzzy parametric uncertain system. A system has parametric uncertainty if there exists a model of it, but the value of its parameters is not exactly known [6]. Many times an estimation of the value of an uncertain parameter is given by an expert (for e.g. 'small' or 'small positive'). In this case, the uncertain parameters can be represented by a fuzzy number, $\tilde{q}_i\in\mathbf{q}$, with membership function,

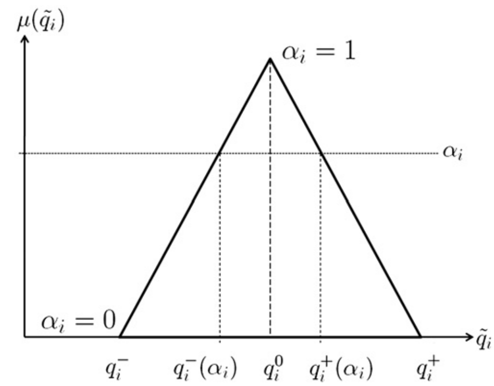

Fig. 1. α-Cut representation of a fuzzy uncertain parameter $\tilde{q}_i$.

$\alpha=\mu(\tilde{q}_i)\in[0,1]$, as shown in Fig. 1. It is to be mention that membership function may be other than triangular membership function. The $\mathbf{q}$ is a universe of discourse for $\tilde{q}_i$. The membership functions $\mu(\tilde{q}_i)$ are unimodal and decrease to the interval endpoints—fact known from the assumption stipulated in the fuzzy approach. Thus, the fuzzy uncertain parameter $\tilde{q}_i$ is defined by an interval, given by its α-cut as

$$q_i(\alpha_i)=[q_i^-(\alpha_i),q_i^+(\alpha_i)] \tag{1}$$

where α_i is the membership level for the q_i, also $q_i^-(.)$ is an increasing function and $q_i^+(.)$ is a decreasing function. Therefore

$$q_i^-(0)=q_i^-,\quad q_i^+(0)=q_i^+,\quad q_i^-(1)=q_i^+(1)=q_i^0 \tag{2}$$

The membership value α_i can be interpreted as the confidence degree in which the value of parameter equals its nominal value. Thus, a value $\alpha_i=1$ indicates the precise knowledge

$$q_i=ker(\tilde{q}_i)=q_i^0 \tag{3}$$

whereas $\alpha_i=0$ represents maximum uncertainty

$$q_i=supp(\tilde{q}_i)=[q_i^-,q_i^+] \tag{4}$$

3. Robust controller design using optimal control approach

Consider a plant with fuzzy parametric uncertainty be represented by the open loop transfer function as

$$\tilde{G}(s,\tilde{p},\tilde{q})=\frac{\tilde{p}_{n-1}s^{n-1}+\cdots+\tilde{p}_1s+\tilde{p}_0}{s^n+\tilde{q}_{n-1}s^{n-1}+\cdots+\tilde{q}_1s+\tilde{q}_0} \tag{5}$$

where $\tilde{p}_i$, and $\tilde{q}_i$, $i=0,1,\ldots,n-1$, represents fuzzy numbers or fuzzy coefficients. If a degree of confidence $\alpha\in[0,1]$ in the coefficients is given then the fuzzy parametric uncertain system in (5) can be represented as an interval system described by the transfer function

$$\tilde{G}(s,\tilde{p}(\alpha),\tilde{q}(\alpha))=\frac{\tilde{p}_{n-1}(\alpha)s^{n-1}+\cdots+\tilde{p}_1(\alpha)s+\tilde{p}_0(\alpha)}{s^n+\tilde{q}_{n-1}(\alpha)s^{n-1}+\cdots+\tilde{q}_1(\alpha)s+\tilde{q}_0(\alpha)} \tag{6}$$

where $\tilde{p}_i(\alpha)$, and $\tilde{q}_i(\alpha)$, $i=0,1,\ldots,n-1$, represents interval numbers or interval coefficients. Thus fuzzy parametric uncertain system (5) is a generalization of an interval system.

The special structure of the system in (5) is a machine tool structure modeled as a second order plant with fuzzy damping and natural frequency [26]. The examples from mechanical engineering, geotechnical engineering, and biomedical engineering can be found in [27].

The fuzzy parametric uncertain system in (6) can be realized in state space representation in the following controllable canonical