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Research Article

Semi-global output feedback stabilization for a class of nonlinear systems using homogeneous domination approach

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ABSTRACT

This paper investigates the problem of semi-global stabilization by output feedback for a class of nonlinear systems using homogeneous domination approach. For each subsystem, we first design an output feedback stabilizer for the nominal system without the perturbing nonlinearities. Then, based on the ideas of the homogeneous systems theory and the adding a power integrator technique, a series of homogeneous output feedback stabilizers are constructed recursively for each subsystem and the closed-loop system is rendered semi-globally asymptotically stable. The efficiency of the output feedback stabilizers is demonstrated by a simulation example.

1. Introduction

In this paper, we consider the problem of semi-global output feedback stabilization for a class of nonlinear systems described by

$$\begin{aligned}\dot{\mathbf{x}}_i &= \mathbf{A}_i\mathbf{x}_i+\mathbf{B}_iu_i+\boldsymbol{\Phi}_i(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m)\\ y_i &= \theta_i x_{i,1}^{p_i},\ i=1,\ldots,m,\end{aligned} \tag{1}$$

where $\mathbf{x}_i=(x_{i,1},x_{i,2},\ldots,x_{i,r_i})^T\in\mathbb{R}^{r_i}$, $u_i\in\mathbb{R}$, $y_i\in\mathbb{R}$ are the states, control input and output for the i-th subsystem, respectively. The nonlinear terms $\boldsymbol{\Phi}_i(\cdot)$'s are uncertain C^1 functions, θ_i's are unknown constants, p_i's are odd integers and

$$\mathbf{A}_i=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\ddots&\vdots\\\vdots&\vdots&\ddots&\ddots&0\\0&0&\cdots&0&1\\0&0&\cdots&0&0\end{bmatrix}_{r_i\times r_i},\quad \mathbf{B}_i=\begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix}_{r_i\times 1}.$$

Due to its practical importance, the problem of output feedback stabilization of (1) has attached a great deal of attention from the nonlinear control community. However, the separation principle in general does not hold for nonlinear systems as shown in [1]. Hence, certain conditions are usually required for the existing global results. Among the different kinds of assumptions, one common condition is that the unmeasurable states cannot be associated with the uncertainties. To deal with the case, a feedback domination design method was proposed under a linear growth condition in [2]. Later [3–5] have solved the problem of global output feedback stabilization under a high-order growth condition by employing the homogeneous domination approach. A unified method has been proposed to deal with nonlinear systems with unknown output function and unknown nonlinearities in [6]. The work [7] investigated the problem of H_∞ decentralized tracking control design with a decentralized observer for interconnected nonlinear systems.

To further relax the aforementioned conditions imposed on global output feedback stabilization, in this paper we pursue a less ambitious control objective, i.e., semi-global output feedback stabilization. It has been shown that the restrictive conditions imposed in these existing global output feedback stabilization results can be further relaxed for semi-global output stabilization. For example, the works [8–10] and [11] achieved the semi-global output feedback stabilization of feedback linearizable (at least partially) systems.

For single-input single-output (SISO) systems, the work [12] explicitly constructed a linear output feedback controller to semi-globally exponentially stabilize a class of nonlinear systems under less restrictive conditions. One novelty of the design method is that the observer and controller are linear and independent of the higher-order nonlinearities and the mismatched uncertainties. In [13], it was shown that semi-global output feedback stabilization was achievable for uniformly completely observable and state

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feedback stabilizable systems. Recently, the theory of homogenous systems [14,15] has been successfully applied the control problem for some nonlinear systems, see for example [3,16,17]. Based on this theory, our recent paper [18] has solved the problem of semi-global finite-time stabilization by output feedback for a class of SISO uncertain nonlinear systems with both higher-order and lower-order terms.

As far as multi-input multi-output (MIMO) systems are concerned, the work [19] presented the semi-global robust stabilization for a class of MIMO nonlinear systems which do not necessarily have a well-defined relative degree nor need to be affine in the control inputs, but still exhibit certain triangular structure. The work [20] considered a class of nonlinear systems in which each subsystem output has a triangular dependence on the states and the overall system has a block triangular form, which extends the existing results to the case that interconnections between the subsystems are allowed. In [21] the problem of semi-global output feedback stabilization was solved by a series of linear output feedback controllers for a class of generalized MIMO uncertain nonlinear systems. However, the observers constructed in [3,21] require the detailed information of $x_{i,1}$ which is currently not available for system (1) since θ_i is an unknown constant. Our recent paper [22] presented the semi-global output feedback stabilization for a class of nonlinear systems with unknown output gains, in which each nominal subsystem has the same homogeneous degree τ. This paper aims to achieve semi-global output feedback stabilization for the uncertain non-triangular system (1) in the presentence of the unknown output gains θ_i, $i=1,\dots,m$ and unknown nonlinearities $\boldsymbol{\Phi}_i$. In addition, each nominal subsystem has a different homogeneous degree τ_i. In the case when the state variables are not measurable but only the output signals are available for feedback, we first design nonlinear homogeneous dynamic compensators. Then we use a block-backstepping, i.e., we treat each subsystem as a block and determine the gains block by block to render the system (1) semi-globally asymptotically stable.

2. Preliminaries

This section contains a definition and several useful lemmas which play important roles in this paper.

Definition 2.1 (*Kawski [14]*). For real numbers $r_i>0, i=1,\dots,n$ and fixed coordinates $(x_1,\dots,x_n)\in\mathbb{R}^n$

- the dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x)=(\varepsilon^{r_1}x_1,\dots,\varepsilon^{r_n}x_n)$, for all $\varepsilon>0$, with r_i being called as the weights of the coordinates. For simplicity, we define dilation weight $\Delta=(r_1,\dots,r_n)$.
- a function $V\in\mathcal{C}(\mathbb{R}^n,\mathbb{R})$ is said to be homogeneous of degree τ if there is a real number $\tau\ge 0$ such that $\forall x\in\mathbb{R}^n\backslash\{0\},\ V(\Delta_\varepsilon(x))=\varepsilon^\tau V(x_1,\dots,x_n)$,
- a vector field $f\in\mathcal{C}(\mathbb{R}^n,\mathbb{R}^n)$ is said to be homogeneous of degree τ if there is a real number $\tau\ge-\min\{r_1,\dots,r_n\}$ such that for $i=1,\dots,n$ $\forall x\in\mathbb{R}^n\backslash\{0\},\ f_i(\Delta_\varepsilon(x))=\varepsilon^{\tau+r_i}f_i(x_1,\dots,x_n)$,
- a homogeneous p-norm is defined as $\|x\|_{\Delta,p}=\left(\sum_{i=1}^n|x_i|^{p/r_i}\right)^{1/p}$ $\forall x\in\mathbb{R}^n$, for a constant $p\ge1$. For simplicity, we choose p=2 and write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Lemma 2.1 (*Rosier [15]*). *Let f be a continuous vector field on $\mathbb{R}^n$ such that the origin is a locally asymptotical stable equilibrium point. Assume that f is homogeneous of degree k with respect to $r=(r_1,\dots,r_n)$, where $r_i>0, i=1,\dots,n$. Then, for any $p\in N^*$ and any $m>p\cdot\max_i\{r_i\}$, there exists a strict Lyapunov function V for system $\dot{x}=f(x), f(0)=0$, which is homogeneous of degree m and of class C^p. As a direct consequence, the time derivative $\dot{V}$ is homogeneous of degree $m+k$.*

Lemma 2.2 (*Bacciotti and Rosier [23]*). *Suppose $V:\mathbb{R}^n\to\mathbb{R}$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following hold*:

(i) *$\partial V/\partial x_i$ is homogeneous of degree $\tau-r_i$* with *r_i being the homogeneous weight of x_i*;
(ii) *there is a constant $\bar{c}$ such that $V(x)\le\bar{c}\|x\|_\Delta^\tau$. Moreover, if $V(x)$ is positive definite, then there exists a constant $\underline{c}>0$, such that $V(x)\ge\underline{c}\|x\|_\Delta^\tau$.*

Lemma 2.3. *For $x\in\mathbb{R}$, $y\in\mathbb{R}$, and $p\ge1$, the following inequalities hold*:

$$|x+y|^p\le 2^{p-1}|x^p+y^p|,$$
$$\left(|x|+|y|\right)^{1/p}\le|x|^{1/p}+|y|^{1/p}\le 2^{(p-1)/p}\left(|x|+|y|\right)^{1/p}. \tag{2}$$

If $p\ge1$ is an odd integer or a ratio of two odd integers,

$$|x-y|^p\le 2^{p-1}|x^p-y^p|,$$
$$|x^p-y^p|\le p|x-y|(x^{p-1}+y^{p-1})\le c|x-y|\,|(x-y)^{p-1}+y^{p-1}| \tag{3}$$

for a positive constant c.

Lemma 2.4. *Suppose c and d are two positive real numbers. Given any real-valued function $\gamma(x,y)>0$, the following inequality holds:*

$$|x|^c|y|^d\le\frac{c}{c+d}\gamma(x,y)|x|^{c+d}+\frac{d}{c+d}\gamma^{-c/d}(x,y)|y|^{c+d}. \tag{4}$$

3. Main results

In this section, we will explicitly construct semi-global output feedback stabilizers for nonlinear systems (1). First, we construct an output feedback stabilizer for the following nominal system:

$$\dot{z}_i=z_{i+1},\ i=1,\dots,n-1,\ \dot{z}_n=v,\ y=\theta z_1^p, \tag{5}$$

where $z=(z_1,z_2,\dots,z_n)^T\in\mathbb{R}^n$, $v\in\mathbb{R}$, $y\in\mathbb{R}$ are the states, control input and output, respectively. p is an odd integer and θ is an unknown constant satisfying the following assumption.

Assumption 3.1. There are two known positive constants $\underline{\theta}$ and $\overline{\theta}$ such that

$$\underline{\theta}\le\theta\le\overline{\theta}. \tag{6}$$

Remark 3.1. In practice, the system parameters in output channel might not be precisely known due to the lack of detailed knowledge of system specifications. Hence in this paper we consider the systems with unknown parameters to encompass more practical systems in applications. On the other hand, the parameters' ranges usually can be estimated. Therefore, in Assumption 3.1, the lower and upper bounds of θ are assumed to be known.

With the help of Assumption 3.1, we are now ready to design an output feedback stabilizer for nominal system (5).

Theorem 3.1. *For any constant $\tau\ge0$, there exist constants d_i and $\beta_i>0, i=1,\dots,n$, such that the following homogeneous output feedback stabilizer*

$$\dot{\hat{z}}_i=\hat{z}_{i+1}-d_i\hat{z}_1^{m_{i+1}},\ i=1,\dots,n-1$$
$$\dot{\hat{z}}_n=v-d_n\hat{z}_1^{m_{n+1}},$$