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Designing a robust minimum variance controller using discrete slide mode controller approach

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ABSTRACT

Designing minimum variance controllers (MVC) for nonlinear systems is confronted with many difficulties. The methods able to identify MIMO nonlinear systems are scarce. Harsh control signals produced by MVC are among other disadvantages of this controller. Besides, MVC is not a robust controller. In this article, the Vector ARX (VARX) model is used for simultaneously modeling the system and disturbance in order to tackle these disadvantages. For ensuring the robustness of the control loop, the discrete slide mode controller design approach is used in designing MVC and generalized MVC (GMVC). The proposed method for controller design is tested on a nonlinear experimental Four-Tank benchmark process and is compared with nonlinear MVCs designed by neural networks. In spite of the simplicity of designing GMVCs for the VARX models with uncertainty, the results show that the proposed method is accurate and implementable.

1. Introduction

The performance of an existing control loop is often measured against some kind of benchmark. Traditional performance measures are used in the case of deterministic disturbances (i.e. set-point changes or sudden load disturbances) such as rise time, settling time, overshoot, offset from set point and integral error criteria. Many designing methods have been proposed from these points of view [1]. However, there are other criteria which are in direct relationship with process performance, product quality, and energy or material consumption, which typically include the variance, or equivalently the standard deviation of the controlled variable or control error. In process control, steady-state regulation is the essential problem. Therefore, responses to load-disturbance are more important than those of set points, as emphasized in [2].

The most widespread stochastic criterion in process control is variance, particularly for regulatory control. The widespread use of variance as a performance criterion is due to the fact that it typically represents product-quality consistency. Bialkowski declared that almost 60% of industrial applications are using minimum variance index [3]. The reduction of variances of many quality variables not only implies improved product quality, but also makes it possible to operate near the constraints for increasing throughput, reducing energy consumption, and saving raw materials [4].

The main questions in dealing with output variance are how much the variance can be decreased and how this aim can be achieved. Minimum Variance Control (MVC) can be regarded as the optimal solution for both questions [5]. The MVC, also referred to as optimal H_2 control and first derived in [6], is the best possible feedback control for linear systems in the sense that it achieves the smallest possible closed-loop output variance [5]. However, the MVC has some main shortcomings that restrict its implementation in real-world applications [4]: (1) this controller needs exact models of the system and disturbance, which are not in hand for many real-world applications; (2) the control signal produced by MVC is so harsh that it can wear out the actuator and, in most cases, actuators cannot respond to control signals; (3) this controller can hardly be employed in nonlinear systems, and (4) it is not a robust controller. These problems are more crucial in MIMO nonlinear cases.

In modeling cases, nonlinear models such as NARMAX [7], neural network [8], and fuzzy models [9,10] have been suggested due to their good accuracy. However, in order to design MVC, the explicit relations between outputs and inputs must be executable. This relation is defined implicitly in the above nonlinear models [11,12]. From a user's point of view, introduced nonlinear models are not user-friendly. In other words, they are not easy-to-use. More specifically, they do not have structures favorable to the

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applications of system control. Consequently, designing MVCs for nonlinear models has been a big issue during the last decade.

Four difficulties have been addressed in previous paragraphs. For overcoming these disadvantages, some procedures have been proposed in this article:

1) The main problem in designing MVC is that it requires an accurate model of the system. In this article, an extension to the multivariate time series analysis [13] method is proposed. The proposed method is capable of being used in designing MVC for nonlinear systems with high accuracy and dealing with the problems mentioned previously. The Linear Vector ARX (VARX) model, which is the extension of ARX or VAR models, is used to design an MVC. As the VARX model can identify the nonlinear property of systems, it can be more accurate than other linear models. In addition, there is no need for pre-filtering the disturbance for identification purposes [14–17]. In the proposed method, the operating sample data of the system is the only requirement for estimating VARX parameters; therefore, there is no need for opening the control loop or stopping the normal operation of the system. This advantage is beneficial for many industrial applications.
2) The second problem in implementing minimum variance controllers is the harsh control signal they often produce. The generalized version of MVC was introduced for solving this disadvantage [18], which is an efficient method for reducing the control signal variance. In this article, generalized minimum variance controller is used for ensuring that the control signal can be implemented. In the proposed method, a VARX model is fitted to the operating data of the system, and a generalized minimum variance controller is designed.
3) The third addressed problem is designing MVC for nonlinear systems. However, the VARX is a linear model. It can be shown that this model can identify some kinds of nonlinear systems (such as four tank systems) with any desired accuracy. Therefore, the controller designed by the VARX is accurate, even for these nonlinear systems. In this article, the result of the proposed method will be compared with nonlinear MVC in which neural network is used for modeling nonlinear systems.
4) The last disadvantage in MVC is lack of robustness [6]. There is uncertainty in almost all existing models; therefore, robustness is an important characteristic of any designed controller. From small gain theory, we know that if the gain of the loop is decreased, the robustness will be increased. However, decreasing gain of the loop will lead to losing the optimality of the controller. Hence, GMVC is more robust and less optimal than MVC. The scope of this article is not limiting the control signal or gain of the control loop. In this article, the approach similar to designing discrete slide mode control will be used to increase robustness of the designed controller.

The organization of this article is as follows: Section 2 provides an explanation of the VARX. In section 3, the robust minimum variance controller by VARX model is explained. Section 4 summarizes the empirical and experimental results on nonlinear four-tank benchmark processes.

2. VARX model

In 1978, the VAR method was introduced for estimating minimum possible variance for MIMO systems [13]. In this method, namely "Multivariate Time Series Analysis Method", Vector AR (VAR) model is fitted to multivariate time series, which are the samples of system output; then, the variance of modeling residuals is considered as minimum possible variance. It is an efficient method that does not need any information on system and controller; however, its estimation of the minimum variance is not accurate. In multivariate time series analysis method, the VAR is defined as [13]

$$\underline{Y}(t)=\sum_{i=1}^{p}\phi_i\underline{Y}(t-i)+\underline{\varepsilon}(t)=\phi_1\underline{Y}(t-1)+\cdots+\phi_p\underline{Y}(t-p)+\underline{\varepsilon}(t)$$

$$\phi_i=\begin{bmatrix}\varphi^i_{11} & \cdots & \varphi^i_{1n}\\ \vdots & \ddots & \vdots\\ \varphi^i_{n1} & \cdots & \varphi^i_{nn}\end{bmatrix},\quad \underline{Y}\left(t\right)=\begin{bmatrix}y_1(t)\\ \vdots\\ y_n(t)\end{bmatrix},$$

$$\underline{\varepsilon}\left(t\right)=\begin{bmatrix}e_{11}(t) & \cdots & e_{n1}(t)\\ \vdots & \ddots & \vdots\\ e_{1n}(t) & \cdots & e_{nn}(t)\end{bmatrix}, \tag{1}$$

VAR cannot be used to design any controllers, as it does not provide any information about exogenous input. VARX is an extension of VAR, which includes exogenous inputs and is defined as

$$\underline{Y}(t)=\sum_{i=1}^{p}\phi_i\underline{Y}(t-i)+\sum_{i=1}^{d}B_i\underline{U}(t-i)+\underline{\varepsilon}(t)=\phi_1\underline{Y}(t-1)+\cdots+\phi_p\underline{Y}(t-p)$$
$$+B_1U(t-1)+\cdots B_dU(t-d)+\underline{\varepsilon}(t)$$

$$\phi_i=\begin{bmatrix}\varphi^i_{11} & \cdots & \varphi^i_{1n}\\ \vdots & \ddots & \vdots\\ \varphi^i_{n1} & \cdots & \varphi^i_{nn}\end{bmatrix},\quad \underline{Y}(t)=\begin{bmatrix}y_1(t)\\ \vdots\\ y_n(t)\end{bmatrix},$$

$$\underline{\varepsilon}(t)=\begin{bmatrix}e_{11}(t) & \cdots & e_{n1}(t)\\ \vdots & \ddots & \vdots\\ e_{1n}(t) & \cdots & e_{nn}(t)\end{bmatrix},\quad B_i=\begin{bmatrix}b^i_{11} & \cdots & b^i_{1m}\\ \vdots & \ddots & \vdots\\ b^i_{n1} & \cdots & b^i_{nm}\end{bmatrix} \tag{2}$$

where p is the model order, n is the number of outputs, m is the number of inputs, d is the order of exogenous inputs terms, $Y(t)$ is the vector of output data, and $\underline{\varepsilon}(t)$ is the model mismatch at sample time t. ϕ_i and B_i are unknown constant parameters of VARX, the values of which must be estimated by a proper algorithm [19].

VARX has many unknown parameters that should be accurately estimated. Suppose the model order (p) is 30, exogenous order is (d) 30, the number of outputs (n) is 5, and number of inputs is 5; the algorithm must be able to estimate $para\text{-}m=n\times n\times p+n\times m\times d=1500$ parameters. Most classical methods, which are used for estimating SISO models, are not able to accurately estimate φ_i matrices; therefore, new methods have been proposed for estimating the value VARX parameters [19–21]. In this article, the Maximum Likelihood (ML) method is used for this purpose [21].

Eq. (2) can be rewritten in short form as follows:

$$\underline{Y}(t)=\sum_{i=1}^{p}\phi_i\underline{Y}(t-i)+\sum_{i=1}^{d}B_i\underline{U}(t-i)+\underline{\varepsilon}(t)=\left(\phi_1q^{-1}+\cdots+\phi_pq^{-p}\right)Y(t)$$
$$+\left(B_1q^{-1}+\cdots+B_dq^{-d}\right)U(t)+\underline{\varepsilon}(t) \tag{3}$$

where q is the backshift (lag) operator.

Assume that

$$\phi(q^{-1})=\phi_1q^{-1}+\cdots+\phi_pq^{-p}$$
$$B(q^{-1})=B_1q^{-1}+\cdots+B_dq^{-d}$$

Then, Eq. (3) can be rewritten as

$$Y(t)=\phi(q^{-1})Y(t)+B(q^{-1})U(t)+\varepsilon(t) \tag{4}$$

The generalized minimum variance controller can be designed using Eq. (4), which is the purpose of Section 3.

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