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Research Article

A numerical investigation for robust stability of fractional-order uncertain systems

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ABSTRACT

This study presents numerical methods for robust stability analysis of closed loop control systems with parameter uncertainty. Methods are based on scan sampling of interval characteristic polynomials from the hypercube of parameter space. Exposed-edge polynomial sampling is used to reduce the computational complexity of robust stability analysis. Computer experiments are used for demonstration of the proposed robust stability test procedures.

1. Introduction

In control theory, increasing trend for fractional-order control system implementation promises superior control performance comparing to integer-order ones. Fractional order calculus (FOC) has been widely studied since discovery of Newton and Leibniz. They proposed an alternative formulation instead of integer-order calculus in the 17th century [1,2]. FOC can be utilized in wide application areas [3] such as physics [4], electrical circuit theory and fractances [5], mechatronic systems [6], signal processing [7], chemical mixing [8], chaos theory [9] etc. Recently FOC has gained a wider attention in control system applications.

Robust control of real systems is one of the substantial problems in control practice due to the fact that parameter uncertainty or model perturbation may cause unstable responses of control systems in real applications, even though the systems were theoretically designed as stable systems in computer simulations. In order to avoid the risk of unstable responses of control systems in applications, stability of systems should be guaranteed within possible ranges of parametric uncertainties and therefore the system should be designed to remain stable against parameter perturbations of uncertain system.

Robust control has been studied in many works for integer [10] and fractional-order systems (FOS) [11,12]. Several works revealed the advantages of FOS [13–15]. Recently, there is a growing effort for stability analysis of linear time invariant (LTI) systems in the literature: Robust stability test method for FOS was presented for fractional-order linear systems with interval uncertainties [16,17]. Petras et al. presented an experimental approach for interval uncertainties for FO-LTI systems [18]. An interval boundary box method for stability testing of the FO-LTI systems with interval uncertainties was presented in [19]. Stabilization analysis of FO-LTI systems based on Lyapunov inequality and linear matrix inequality (LMI) was proposed in [20–22]. Robust stability check based on four Kharitonov's polynomials was presented for commensurate order LTI fractional-order system in [16,17,23]. A sufficient and necessary condition for the robust asymptotical stability of fractional-order interval systems with the order satisfying $0<\alpha<1$ was demonstrated in [24]. Another study presented a solution for robust stability of time-varying systems by using linear fractional representation [25]. Lim et al. proposed a method for asymptotical stabilization of fractional order linear systems subject to input saturation provided by using Gronwall–Bellman lemma [26]. In addition, bounded-input bounded-output (BIBO) stability of a large class of neutral type fractional delay systems is investigated in [27].

Stability analysis mostly depends on calculating eigenvalues of state equations of LTI systems in the literature [28,29]. Stability check via root locus is a useful technique for closed loop control system design practice. Hence, extending the root locus based stability check procedures for the fractional order systems involving interval uncertainty of polynomial orders and coefficients may yield new horizons in fractional order control study.

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This study is devoted for numerical investigation of robust stability of fractional-order closed-loop control systems with parameter uncertainty. The study also considers both coefficient and order uncertainty cases of FOS. The study benefits from $s=v^m$ mapping in order to turn fractional-order characteristic polynomials of control systems into expanded integer-order polynomials [12]. Two numerical methods based on Radwan et al. procedure [12] are discussed for the stability analysis of interval polynomial families. The first method uses scan sampling of test polynomials from intervals of uncertain parameters, and consequently, computational complexity of the method exponentially grows with respect to the spatial sampling density of parameter space. The second method based on Edge Theorem samples edge polynomials from parameter space, and hence considers the boundary line of root plane and therefore it considerably decreases the computational complexity.

The Edge Theorem has been successfully utilized in the robust stability analysis of integer-order systems. This theorem establishes the fundamental property that the root space boundary of a polytopic family of polynomials is contained in the root locus evaluated along the exposed edges [10]. In a previous work, Kang et al. briefly showed a diligent use of Edge Theorem for robust stability analysis of commensurate fractional-order interval polynomials [27]. Thanks to Edge Theorem that effectively and constructively reduce the problem of determining the root space under multiple parameter uncertainty to a set of one-parameter root locus problem [10].

Motivation of this study comes from root region investigation of sampled interval polynomials according to Radwan procedure for the computer aided robust stability analysis of fractional-order control systems with interval uncertainty. This study contributes to numerical investigation of robust stability of fractional-order closed-loop control systems.

2. Methodology

2.1. Theoretical background

Fig. 1 shows general representation of a closed loop control structure. The characteristic polynomial of this system can be given as,

$$\Delta(s)=1+C(s)G(s)H(s) \tag{1}$$

Let us define a fractional-order characteristic polynomial of this control system in the form of

$$\Delta(s)=\sum_{i=1}^{n} a_i s^{\alpha_i}, \tag{2}$$

where, the parameters $a_i \in R$ are the polynomial coefficients for $i=1, 2, 3, \ldots, n$ and $\alpha_i = c_i/d_i$ are the fractional orders of polynomial [3]. Here, c_i and d_i are the positive integer numbers. The parameter α_i is assumed to satisfy the condition of $\alpha_1=0$ for a constant term of polynomials and $\alpha_{i+1}>\alpha_i$.

One of the common ways to decide the stability of a FO-LTI system is to apply $s=v^m$ mapping to the system transfer function [19,20]. This mapping simply transforms fractional-order polynomials to expanded degree integer-order polynomials, and thus it reduces a fractional-order system stability analysis problem to stability analysis of an integer-order system in the Riemann sheets. This allows the assessment of the stability of fractional-order polynomials according to root locus analysis in their root spaces. Assuming $\alpha=1/m$, m number of Riemann sheets are defined in complex root plain ($s=|s|e^{j\varphi}$) as $(2k+1)\pi<\phi<(2k+3)\pi$, where $k=-1, 0, \ldots, m-2$ [30].

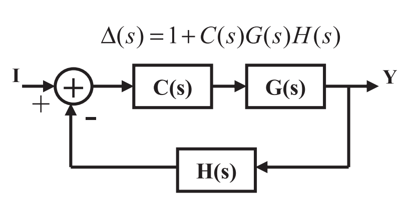

Fig. 1. Block diagram of a closed loop control system.

Applying transformation $s=v^m$ for a positive integer $m>1$, expanded integer-order form of the characteristic polynomial in Eq. (1) can be written as,

$$\Delta^m(v)=\sum_{i=1}^{n} a_i v^{\beta_i}, \tag{3}$$

where, $\Delta^m(v)$ represents $m-th$ order expanded characteristic polynomial of the fractional-order control system and $\beta_i = m\alpha_i$ are the expanded integer orders. In order to expand a fractional-order characteristic polynomial to a minimal degree integer-order characteristic polynomial, m expansion order can be chosen as the least common multiplier (LCM) of $d_1, d_2, d_3, \ldots, d_n$ that transforms the fractional orders into minimal integer orders [31]. In this case, the degree of the polynomial can be given as $\xi=\max\{\beta_1, \beta_2, \ldots, \beta_n\}$ [17] and this integer-order polynomial with degree of ξ is referred to expanded integer-order representation of the fractional-order system.

An expanded integer order polynomial of a fractional-order control system has ξ roots on the Riemann surface [20]. Let denote these roots as $v_1, v_2, v_3, \ldots, v_\xi$. Roots residing in the phase range of $\varphi \in (-\pi/m, \pi/m)$, which coincides to the first Riemann sheet, are physically meaningful and these roots of the expanded integer order system are used for the stability analysis of control systems [32].

Root location of expanded integer-order characteristic polynomial ($\Delta^m(v)$) in the root space was effectively used to decide stability of fractional order LTI systems [12,16,32,33]. The systematic approach in the robust stability analysis is provided by Radwan et al. [12]. Radwan procedure can be summarized as,

- ⁎ Calculate absolute values of root phases ($|\phi_v|=|\arg(v_r)|$), $r\in[1,\xi]$) of expanded integer order system polynomial in the first Riemann sheet ($\phi \in (-\pi/m, \pi/m)$).
- ⁎ If the values of root phases are in the range of $\pi/2m < |\phi_v| < \pi/m$ in the first Riemann sheet, this system is stable.
- ⁎ If the values of root phases are equal to $\pi/2m$, the system oscillates.
- ⁎ Otherwise, the system is unstable.

As a special case of $s=v^m$ mapping for $m=1$, the principle Riemann surface refers to the open left half of the complex plane. This half plane is stability region of integer order characteristic polynomials. The stability condition for integer-order systems can be written as $\pi/2<|\phi_v|<\pi$ for $m=1$. Fig. 2 illustrates stability regions according to absolute root phase values ($|\phi_v|=|\arg(v_r)|$) in the root spaces for $m=1$ and $m>1$ cases.

2.2. Robust stability analysis for fractional-order systems with uncertain coefficients

For the robust stability analysis of the closed loop control system containing interval uncertainty in the predefined finite ranges, it is convenient to define uncertain characteristic polynomials in the form of an interval polynomial as follows,

$$\Delta_u(s)=\sum_{i=1}^{n} [\underline{a}_i\ \overline{a}_i] s^{\alpha_i}, \tag{4}$$