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Research Article

Performance limitations in the tracking and regulation problem for discrete-time systems ☆

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ABSTRACT

In this paper, the optimal tracking and regulation performance of discrete-time, multi-input multi-output, linear time-invariant systems is investigated. The control signal is influenced by the external disturbance, and the output feedback is subjected to an additive white Gaussian noise (AWGN) corruption. The tracking error with channel input power constraint and the output regulation with control energy constraint are adopted as the measure of tracking and regulation performance respectively, which can be obtained by searching through all stabilizing two-parameter controllers. Both results demonstrate that the performance is closely related to locations and directions of the nonminimum phase zeros, unstable poles of the plant and may be badly degraded by external disturbance and AWGN.

1. Introduction

In recent years, there has been growing attention devoted to the study of intrinsic performance limits achievable by feedback control [1–5]. One of the well-studied problems is optimal tracking [6–11]. It is known that in general the minimal tracking error depends on the nonminimum phase zeros, unstable poles and time delays in the plant [12]. These results are obtained based on the conventional assumption that controller and plant communicate information in an ideal manner.

Lately, increasing interest was devoted to the study of control systems in which non-ideal data transmission occurs. Significant research attention has been paid to networked control systems (NCSs) due to their significant advantages [13,14]. In another aspect, the discrete systems are widely used in the actual environment, such as computer control systems and sample control systems. Actually, NCSs are a typical discrete-time systems [15,16]. In NCSs, due to long distance communication channels, and thus, issues such as time delay [17], noise [18], quantization [19], packet dropouts [20] have to be considered. The optimal $\mathcal{H}_2$ performance of networked control system can be found in [21], and the analysis has been done for the tracking control problems [22–25]. It is shown in paper [22] that the additive white Gaussian noise imposes unavoidable limitations on achievable performance in tracking a Brownian motion random process. It also points out that two-parameter controller structure can improve the tracking performance. However, in [22,26], the disturbance rejection has not been considered in tracking problem, whereas, the disturbance often appears in the actual control system. An optimality-based framework for addressing the problem of nonlinear non-quadratic hybrid control for disturbance rejection is studied in paper [27].

The previous results provide some useful information about the relationship between the control performance and the plant characteristics. Nevertheless, it is useful to note that channel input is often required to satisfy the power constraint, which may arise either from electronic hardware limitations or regulatory constraints introduced to minimize interference to other communication system users. Moreover, the bandwidth of the plant may constrain the tracking accuracy when its input energy is finite, and control limitation due to the plant bandwidth is frequently encountered in practical designs, but rarely seems to have been characterized analytically. Based on these considerations, this paper firstly studies the optimal tracking problems concerning MIMO linear time-invariant (LTI) feedback control systems with input disturbance and feedback channel noise, and under channel input power constraint. Furthermore, regulation problem under control energy

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constraint is also investigated. Thus, one contribution of this paper is that new tracking performance index combining with channel input power constraint and regulation performance combining with control energy constraint are proposed. Then two explicit expressions on optimal tracking and regulation performance are obtained by using some factorization of transfer function matrix and Youla parameterization of controllers, which is the main difference from [28]. On the other hands, we quantitatively reveal how the tracking and regulation performance are affected by input disturbance and feedback channel noise. Both the above results will be very helpful for the design of NCSs including optimal controllers and communication channel.

The remainder of this paper is organized as follows. In Section 2, we define some notations and introduce the Youla parameterization of all stabilizing controllers, We then proceed in Section 3 to formulate and solve the problem of optimal tracking under channel input power constraint, and optimal regulation under control energy constraint in Section 4. An illustrative example is given in Section 5 to show effectiveness of the obtained theoretic results. Finally, Section 6 draws conclusion.

2. Preliminaries

We begin by summarizing briefly some notations used throughout this paper. For any complex number z, we denote its complex conjugate by $\bar{z}$. For any vector u, we denote its conjugate transpose by u^H, and its Euclidean norm by $\|u\|$. All the vectors and matrices involved in the sequel are assumed to have compatible dimensions, and for simplicity their dimensions will be omitted. Let the open unit disc be denoted by $D{:=}\{z \in C : |z| < 1\}$, the closed unit disc by $\overline{D}{:=}\{z \in C : |z| \le 1\}$, the unit circle by $\partial D{:=}\{z \in C : |z| = 1\}$, and the complement of $\overline{D}$ by $\overline{D}^c{:=}\{z \in C : |z| > 1\}$. Moreover, let $\|\cdot\|$ denotes the Euclidean vector norm and $\|\cdot\|_F$ the Frobenius norm $\|G\|_F^2{:=}\text{tr}(G^H G)$, tr the trace of the matrix. Define

$$\mathcal{L}_2{:=}\left\{G : G(z) \text{ measurable in } \partial D,\ \|G\|_2^2{:=}\frac{1}{2\pi}\int_{-\pi}^{\pi}\|G(e^{j\theta})\|_F^2\, d\theta < \infty\right\}.$$

Then, $\mathcal{L}_2$ is a Hilbert space with an inner product

$$\langle F, G\rangle{:=}\frac{1}{2\pi}\int_{-\pi}^{\pi}\text{tr}(F^H(e^{j\theta})G(e^{j\theta}))\, d\theta.$$

Next, define

$$\mathcal{H}_2{:=}\left\{G : G(z) \text{ analytic in } \overline{\mathrm{D}}^c,\ \|G\|_2^2{:=}\sup_{r>1}\frac{1}{2\pi}\int_{-\pi}^{\pi}\|G(re^{j\theta})\|_F^2\, d\theta < \infty\right\},$$

and

$$\mathcal{H}_2^{\perp}{:=}\left\{G : G(z) \text{ analytic in D},\ \|G\|_2^2{:=}\sup_{r<1}\frac{1}{2\pi}\int_{-\pi}^{\pi}\|G(re^{j\theta})\|_F^2\, d\theta < \infty\right\}.$$

It is well known that $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$ are subspaces and form an orthogonal pair of $\mathcal{L}_2$. Similarly, define $\mathcal{H}_\infty$ as the space of all complex valued matrix functions which are bounded and analytic in D^c, and $\mathcal{RH}_\infty$ the space of all rational matrix functions in $\mathcal{H}_\infty$.

Next we introduce some important factorizations that will be frequently used. For the rational right-invertible transfer function matrix P, let its right and left coprime factorizations be given by

$$P = NM^{-1} = \tilde{M}^{-1}\tilde{N}, \tag{1}$$

where $N, \tilde{N}, \tilde{M}, M \in \mathcal{RH}_\infty$, and satisfy the double Bezout identity

$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix}\begin{bmatrix} M & Y \\ N & X \end{bmatrix} = I, \tag{2}$$

for some $X, \tilde{X}, \tilde{Y}, Y \in \mathcal{RH}_\infty$. It is well known that all the stabilising two parameter compensators K can be characterized by the Youla parameterization

$$\mathcal{K}{:=}\{K : K = [K_1\ K_2] = (\tilde{X} - R\tilde{N})^{-1}[Q\tilde{Y} - R\tilde{M}],\ Q, R \in \mathcal{RH}_\infty\}. \tag{3}$$

It is also well known that a nonminimum phase transfer function matrix could be factorized into a minimum phase part and an all-pass factor [9]. Denote $s_i \in \overline{D}^c$, $i = 1, \ldots, N_z$ as the nonminimum phase zeros of $P(z)$, which are also the nonminimum phase zeros of $N(z)$, thus $N(z)$ can be factorized as

$$N(z) = L(z)N_m(z), \tag{4}$$

where $N_m(z) \in \mathcal{RH}_\infty$ and

$$L(z) = \prod_{i=1}^{N_z} L_i(z), \quad L_i(z) = \frac{1-\bar{s}_i}{1-s_i}\frac{z-s_i}{1-\bar{s}_i z}\eta_i\eta_i^H + U_iU_i^H, \tag{5}$$

with η_i being an unitary vector $\|\eta_i\| = 1$, which can be sequentially determined from the zero direction vectors of P, and U_i being a matrix such that $\eta_i\eta_i^H + U_iU_i^H = I$. Similarly, denote $p_k \in \overline{D}^c$, $k = 1, \ldots, N_p$ as the unstable poles of $P(z)$, then $\tilde{M}(z)$ can be factorized as

$$\tilde{M}(z) = \tilde{M}_m(z)\tilde{B}(z),$$

where $\tilde{M}_m(z) \in \mathcal{RH}_\infty$, and

$$\tilde{B}(z) = \prod_{i=1}^{N_p} \tilde{B}_k(z), \quad \tilde{B}_k(z)\frac{z-p_k}{1-\bar{p}_k z}\tilde{\omega}_k\tilde{\omega}_k^H + \tilde{W}_k\tilde{W}_k^H,$$

where the unitary vector $\tilde{\omega}_k$ can be computed analogously from the pole direction vectors of P, and $\tilde{W}_k$ being a matrix such that $\tilde{\omega}_k\tilde{\omega}_k^H + \tilde{W}_k\tilde{W}_k^H = I$. If W is a real diagonal matrix, then $\tilde{M}W$ can be factorized as

$$\tilde{M}(z)W = M_m(z)B(z),$$

where $M_m(z) \in \mathcal{RH}_\infty$, and

$$B(z) = \prod_{k=1}^{N_p} B_k(z), \quad B_k(z) = \frac{z-p_k}{1-\bar{p}_k z}\omega_k\omega_k^H + W_kW_k^H,$$

with ω_k being an unitary vector

$$\omega_k = \frac{W^{-1}\tilde{\omega}_k}{\|W^{-1}\tilde{\omega}_k\|},$$

and W_k is matrix such that $\omega_k\omega_k^H + W_kW_k^H = I$.

In this paper the reference signal r as shown in Fig. 1 is stochastic signal, which can often be applied in certain environmental monitoring applications or in business and economic application, thus the research of stochastic signal is necessary. Denote

$$r = r(k) = [r_1(k), r_2(k), \ldots, r_m(k)]^T,$$

and the disturbance signal

$$d = d(k) = [d_1(k), d_2(k), \ldots, d_m(k)]^T.$$

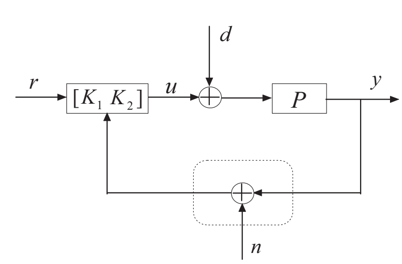

Fig. 1. The two-parameter networked feedback tracking control system.