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Research Article

Further results on delay-range-dependent stability with additive time-varying delay systems

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ABSTRACT

In this paper, new conditions for the delay-range-dependent stability analysis of time-varying delay systems are proposed in a Lyapunov–Krasovskii framework. Time delay is considered to be time-varying and has lower and upper bounds. A new method is first presented for a system with two time delays, integral inequality approach (IIA) used to express relationships among terms of Leibniz–Newton formula. Constructing a novel Lyapunov–Krasovskii functional includes information belonging to a given range; new delay-range-dependent criterion is established in term of linear matrix inequality (LMI). The advantage of that criterion lies in its simplicity and less conservative. This paper also presents a new result of stability analysis for continuous systems with two additive time-variant components representing a general class of delay with strong application background in network-based control systems. Resulting criteria are then expressed in terms of convex optimization with LMI constraints, allowing for use of efficient solvers. Finally, three numerical examples show these methods reducing conservatism and improving maximal allowable delay.

1. Introduction

Delay systems and especially its asymptotic stability have been thoroughly studied since several decades [1–26] and references therein. Study of the delay phenomenon is motivated by its applied aspect. Indeed, many processes include dead-time phenomena in their dynamics such as manufacturing system, turbojet engine, telecommunication, economic system and chemical engineering system. It is generally regarded as a main source of instability and poor performance. They have mainly employed a Lyapunov functional to derive delay-dependent criteria; resulting methods are generally classified into two categories: discretized-Lyapunov-functional method and methods based on the Leibniz–Newton formula. It is difficult to extend the former type [7] to solve the synthesis problem for a control system; the latter type generally requires system transformation. Four basic fixed model transformations are been presented [5], but they all entail a certain degree of conservativeness that leaves room for further investigation.

Recently, in order to reduce the conservatism, a free weighting matrix method was proposed [8,9,24] to evaluate delay-dependent stability wherein bounding techniques on some cross product terms are not involved. He et al. [9] and Wu et al. [24] devised a new method that employed free weighting matrices to express relationships between terms of the Leibniz–Newton formula [16,21,23]. This overcomes conservativeness of methods involving a fixed model transformation. Ramakrishnan and Ray [20] have devised mathematical techniques based on congruent transformation to eliminate the free-weighting matrices in existing stability criteria without sacrificing conservatism [26]. Hence computational burden involved in solving stability criteria using standard numerical packages becomes minimal. Yet when estimating the upper bound of the derivative of Lyapunov functional for systems with time-varying delay, there is room for investigation. Most of the known results on this problem are derived assuming only that the time-varying delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative: $0 \le h_d < 1$ [7,23]. Such restriction is very conservative and of no practical significance. On the other hand, range of time-varying delay considered in [5,9,20,25] is from 0 to an upper bound. Unlike previous methods, the upper bound of delay derivative is taken into account consideration, even if greater than or equal to 1 [8,17,25]. In practice, time-varying interval delay is often encountered: range of delay varies in an interval for which the lower bound is not restricted to 0. In this case, the stability criteria in [5,9,20,24] are conservative, since they do not take into account information on the lower bound of delay. Many researchers have realized that many free variables introduced in the free weighting matrices method will complicate system analysis, thereby yielding

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significant increase in computational demand [2,19]. In the case of time varying delay, results are much scarcer and proposed methodology often conservative. Up to now, to the best of the authors' knowledge, few results have been reported in literature for time delay systems with time-varying interval delay. Still, estimating upper bound of Lyapunov functional derivative for time-varying systems with different time delay range leaves room for investigation.

Closely related to time-delay systems, network-based control has emerged as a topic of significant interest in the control community. It is well known that in many practical systems, physical plant, controller, sensor and actuator are difficult to locate at the same place, meaning signals must be transmitted from one place to another. In modern industrial systems, these components are often connected over network media (typically digital band-limited serial communication channels), giving rise to so-called networked control systems (NCSs). Therefore, NCSs receive more and more attention and become more and more popular in many practical applications in recent years [3,6,10,12,20]. In an NCS, the most significant feature is network-induced delay, chiefly caused by limited bits rate of communication channels, by node waiting to send out a packet via busy channel, or by signal processing and propagation. Existence of signal transmission delay generally exerts negative effects on NCS stability and performance. Yet the relation between time-varying delay and its upper bound is taken into account when estimating upper bound of the derivative of Lyapunov functional, methods in [3,6,10,12,17,20] are rather conservative, and there remain chance to improve results. This motivates researchers to study the delay-dependent stability for time-varying systems with two additive delay components.

This paper studies the stability problem for systems with time-varying delay in a range by choosing an appropriate Lyapunov functional. Based on such stability conditions derived via Lyapunov–Krasovskii functional combining with LMI techniques and an integral inequality approach to express relationships among terms of Leibniz–Newton formula, delay-range-dependent stability criteria emerge. Less conservative result is obtained by considering some useful terms when estimating the upper bound of the derivative of Lyapunov functional and introducing additional terms into the proposed Lyapunov function that includes information of the range. Less conservative stability criteria are established for systems with two additive time-varying delay components. Numerical examples show proposed criteria effectively improving over some existing results in the literature.

2. Stability analysis

Consider the following linear system with time-varying delay

$$\dot{x}(t)=Ax(t)+Bx(t-h(t)) \quad t>0 \tag{1a}$$

$$x(t)=\varphi(t), \quad t\in[-h_2,0] \tag{1b}$$

where $x(t)\in R^n$ is the state vector; $A,B\in R^{n\times n}$ are constant matrices; $\varphi(t)$ is a continuously real-valued initial function on $[-h_2,0]$. $h(t)$ is a continuous time-varying function that satisfies

$$0\le h_1\le h(t)\le h_2 \tag{2}$$

and

$$\dot{h}(t)\le h_d \tag{3}$$

where h_1, h_2, and h_d are positive constants. Note h_1 may not equal 0.

Our goals are to establish a sufficient condition on delay-range-dependent stability and to give estimates of h_d, h_1 and h_2. The following technical Lemma 1 of integral inequality approach will be used in the sequel.

Lemma 1. *(Liu [16,17])*

If there exist symmetric positive-definite matrix $X_{33}>0$ and arbitrary matrices X_{11}, X_{12}, X_{13}, X_{22} and X_{23} such that

$$X=\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33}\end{bmatrix}\ge 0 \tag{4a}$$

then we obtain

$$-\int_{t-h(t)}^{t}\dot{x}^T(s)X_{33}\dot{x}(s)ds\le$$

$$\int_{t-h(t)}^{t}[x^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(s)]\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s)\end{bmatrix}ds \tag{4b}$$

This paper constructs a new Lyapunov functional that contains information of the lower bound of delay h_1 and upper bound h_2.Theorem 1 presents delay-range-dependent result in terms of LMIs and expresses relationships between the terms of the Leibniz–Newton formula.

Theorem 1. *Given scalars $h_1>0$, $h_2>0$ and $h_d>0$, system (1) with time-varying $h(t)$ satisfying (2) and (3) is asymptotically stable if there exist symmetric positive-definite matrices $P=P^T>0$, $Q_i=Q_i^T>0$, $R_i=R_i^T>0$ $(i=1,2,3)$ and positive semi-definite matrices*

$$X=\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33}\end{bmatrix}\ge 0,\quad Y=\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33}\end{bmatrix}\ge 0,$$

$$Z=\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12}^T & Z_{22} & Z_{23} \\ Z_{13}^T & Z_{23}^T & Z_{33}\end{bmatrix}\ge 0,$$

such that the following LMIs hold

$$\Xi=\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ \Xi_{12}^T & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\ \Xi_{13}^T & \Xi_{23}^T & \Xi_{33} & 0 & 0 \\ \Xi_{14}^T & \Xi_{24}^T & 0 & \Xi_{44} & 0 \\ \Xi_{15}^T & \Xi_{25}^T & 0 & 0 & \Xi_{55}\end{bmatrix}<0 \tag{5a}$$

and

$$R_1-X_{33}\ge 0 \tag{5b}$$

$$R_2-Y_{33}\ge 0 \tag{5c}$$

$$R_3-Z_{33}\ge 0 \tag{5d}$$

where

$$\Xi_{11}=A^TP+PA+Q_1+Q_2+Q_3+h_1X_{11}+X_{13}+X_{13}^T+h_2Y_{11}+Y_{13}+Y_{13}^T,$$

$$\Xi_{12}=PB, \Xi_{13}=h_1X_{12}-X_{13}+X_{23}^T,\quad \Xi_{14}=h_2Y_{12}-Y_{13}+Y_{23}^T,$$

$$\Xi_{15}=h_1A^TR_1+h_2A^TR_2+h_{21}A^TR_3,$$

$$\Xi_{22}=-(1-h_d)Q_3+h_{21}Z_{22}-Z_{23}-Z_{23}^T+h_{21}Z_{11}+Z_{13}+Z_{13}^T,$$

$$\Xi_{23}=h_{21}Z_{12}^T-Z_{13}^T+Z_{23}, \Xi_{24}=h_{21}Z_{12}-Z_{13}+Z_{23}^T,$$

$$\Xi_{25}=h_1B^TR_1+h_2B^TR_2+h_{21}B^TR_3,$$

$$\Xi_{33}=-Q_1+h_1X_{22}-X_{23}-X_{23}^T+h_{21}Z_{11}+Z_{13}+Z_{13}^T,$$

$$\Xi_{44}=-Q_2+h_2Y_{22}-Y_{23}-Y_{23}^T+h_{21}Z_{22}-Z_{23}-Z_{23}^T,$$

$$\Xi_{55}=-h_1R_1-h_2R_2-h_{21}R_3, h_{21}=h_2-h_1.$$