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Research Article

A new optimal sliding mode controller design using scalar sign function

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ABSTRACT

This paper presents a new optimal sliding mode controller using the scalar sign function method. A smooth, continuous-time scalar sign function is used to replace the discontinuous switching function in the design of a sliding mode controller. The proposed sliding mode controller is designed using an optimal Linear Quadratic Regulator (LQR) approach. The sliding surface of the system is designed using stable eigenvectors and the scalar sign function. Controller simulations are compared with another existing optimal sliding mode controller. To test the effectiveness of the proposed controller, the controller is implemented on an aluminum beam with piezoceramic sensor and actuator for vibration control. This paper includes the control design and stability analysis of the new optimal sliding mode controller, followed by simulation and experimental results. The simulation and experimental results show that the proposed approach is very effective.

1. Introduction

Sliding mode controllers are known for their robustness and stability. The concept of Sliding Mode Control (SMC) has received much attention in the past few decades. The concept of sliding mode was first proposed by Utkin [1] as a Variable Structure System (VSS) controller and showed that a sliding mode could be achieved by changing the controller structure. In the sliding mode controller, the system state trajectory is forced to move along a chosen stable manifold, called the sliding manifold, in the state space. The sliding manifold is always chosen in such a manner that derived control law guarantees the system stability. Young et al. [2] presented a guide for control engineers to design different sliding mode controllers.

Since SMC's inception, many different techniques have been proposed for choosing the sliding manifold, sometimes referred to as a sliding mode. Xu et al. [3] proposed an optimal sliding mode controller to solve the infinite time optimal control problem. An LQR approach was used to calculate the optimal gain for the sliding mode controller and to deal with uncertainties stochastically. Laghrouche et al. [4] proposed another higher order sliding mode control based on an optimal LQR approach. Many complex hybrid sliding mode controller structures also have been proposed in association with other techniques, such as adaptive control techniques and fuzzy control techniques [5–10]. These techniques ensure asymptotical stability and the reduction of chattering. However, most of these hybrid controllers require complex implementation algorithms.

Nikkah et al. [11] proposed a novel method based on nonlinear predictive control to design optimal linear sliding surfaces for control of under-actuated systems. In this method, the proposed sliding surface is a combination of the classic linear surface and an adaptive time varying linear component. In their approach, even if optimization of the system is not feasible, the controller has to be implemented, making this approach a bit cumbersome. On the other hand, Niu et al. [12] proposed an improved sliding mode control algorithm for discrete time systems. They proposed a new reaching law for the sliding surface; however, the proposed reaching law still has some conditions to satisfy. Shieh et al. [13] proposed a robust sliding mode control approach for magnetic levitation systems. In their approach, integral sliding mode control with a robust optimal approach was developed to achieve high performance in position tracking. Li et al. [14] implemented a PD-sliding mode hybrid controller to control the speed of permanent magnet synchronous motor robustly.

Sliding Mode control algorithm has been used extensively by researchers in vibration control of flexible structures. Song et al. [15] has implemented sliding mode based controller to suppress the vibrations of a flexible beam. Later, Gu et al. [16] has implemented a fuzzy logic based adaptive sliding mode controller for

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controlling the vibration of flexible aerospace structures. Li et al. [17] proposed and successfully implemented a dynamic neural network based adaptive fuzzy sliding mode approach for non-linear structural vibration suppression.

The main objective of the sliding mode control can be summarized as to derive the state from an arbitrary initial state to the origin along a user-specified trajectory via a robust, optimal and less chattering controller. The developed sliding mode controller should be able to optimally tracking and robustly staying in the user-specified sliding surface. In addition, it should be able to reduce the chattering phenomenon, a non-smooth functional behavior, so that it would not damage the control actuator and excite the undesirable unmolded dynamics. The nice properties of the scalar/matrix sign function [18] enable to develop a sliding mode controller to achieve the afore-mentioned objective. The afore-mentioned sign function has the nice properties that it is able to replace the non-smooth signum function by a smooth and differentiable continuous-time function [19,20] and able to carry out the separation of matrix eigenvalues/eigenvectors for structural decomposition of large-scale system matrices [21] by means of numerically fast and stable algorithms [22,23].

In this paper, a new optimal sliding mode control strategy is proposed based on an optimal LQR approach. First, a robustly stable system is designed using the LQR approach by optimally placing the poles of the system in a vertical strip of the left half of the s-plane. The sliding surface will be chosen with the stable eigenvectors and the controller is designed using scalar and matrix sign functions [18], ensuring the satisfaction of the conditions in [12] and convergence of states to the desired sliding surface. As constrained optimization, the states of the robustly stable system track the desired sliding surface. The desired eigenvectors are calculated using the matrix sign function to avoid the complex eigenvectors of system. The paper includes the development of an optimal control algorithm incorporating the scalar and matrix sign-function method. A stability analysis of the proposed method is also provided. The developed control algorithm is simulated as an example and compared with the classical sliding mode control technique [24]. The sign function is used to calculate the eigen-vectors which are used to design the stable sliding surface and to replace the signum function in the sliding mode control algorithm. The novel strategy is later implemented as a solution to a vibration control problem. The experimental results are quite encouraging and are presented in the paper.

2. Introduction to sign function

The matrix sign function was first introduced by Roberts [25] and was applied to solve Lyapunov's equation, the algebraic Riccati equation and the model reduction problem. A comprehensive survey on the matrix sign function and its applications can be found in [26]. The scalar sign function can be defined over the complex plane minus the imaginary axis [25] as

$$\mathrm{sign}(z)=\begin{cases}1 & \text{if } \mathrm{Re}(z)>0,\\ \text{undefined} & \mathrm{Re}(z)=0,\\ -1 & \text{if } \mathrm{Re}(z)<0,\end{cases} \tag{1}$$

where $z\in C\backslash C^0$ (i.e., $C^+\cup C^-$), and C^+, C^- and C^0 denote the open right-half complex plane, the open left-half complex plane and the imaginary axis, respectively and $\mathrm{Re}(\cdot)$ denotes the real part of $(\cdot)$.

In [18], an alternative form to represent the scalar sign function was given by

$$\mathrm{sign}(z)=\begin{cases}\frac{g(z)}{z} & \text{if } \mathrm{Re}(z)>0,\\ \text{undefined} & \mathrm{Re}(z)=0,\\ \frac{g(z)}{z} & \text{if } \mathrm{Re}(z)<0\end{cases} \tag{2}$$

where $g(z)=\sqrt{z^2}$ is the principal square-root of the complex value z^2 and can be expressed as

$$g(z)=\begin{cases}z & \text{if } \mathrm{Re}(z)>0,\\ -z & \text{if } \mathrm{Re}(z)<0.\end{cases} \tag{3}$$

again, $\mathrm{Re}(z)=0$ is not included in the definition. In [18], it is shown that $g(z)$ can be expressed by the continued fraction expansion form

$$g(z)=\sqrt{z^2}=1+\frac{z^2-1}{2+\frac{z^2-1}{2+\cdots}}, \tag{4}$$

where $z\in\mathbf{C}^+\cup\mathbf{C}^-$. Also, it has been shown that the jth truncation of the continued fraction expansion of $g(z)$ can be written as

$$g_j(z)=z\frac{(1+z)^j-(1-z)^j}{(1+z)^j+(1-z)^j},\quad \text{for}\quad j=1,2\ldots \tag{5}$$

Substituting (5) into (2) gives an exact expression of the scalar sign function as

$$\mathrm{sign}(z)=\frac{g(z)}{z}=\lim_{j\to\infty}\frac{g_j(z)}{z}=\lim_{j\to\infty}\mathrm{sign}_j(z), \tag{6a}$$

where

$$\mathrm{sign}_j(z)=\frac{(1+z)^j-(1-z)^j}{(1+z)^j+(1-z)^j}, \tag{6b}$$

and $\mathrm{sign}_j(z)$ is the jth-order approximation of the scalar sign function in (3). $\mathrm{sign}_j(z)$ can be then expressed as

$$sign_j(z)=\begin{cases}+1, & \text{for } z>0 \text{ and } j\to\infty,\\ 0, & \text{for } z=0,\\ -1, & \text{for } z<0 \text{ and } j\to\infty.\end{cases} \tag{7a}$$

As shown above, $\mathrm{sign}_j(z)$ is a continuous and differentiable function that includes $z=0$ in the definition. Generally, even values of j should be used. For even values of j, $\mathrm{sign}_j(z)\in[-1,1]$, and for odd values of j, $\mathrm{sign}_j(z)\notin[-1,1]$. In addition, the absolute-value function of a variable $z$can be uniquely expressed by the sign function as

$$|z|\cong z\,\mathrm{sign}_j(z). \tag{7b}$$

The scalar sign function derivation can also be extended to a matrix sign function. In [18], the matrix sign function was defined as

$$\mathrm{sign}_j(\mathbf{Z})=[(\mathbf{I_n}+\mathbf{Z})^{\mathbf{j}}-(\mathbf{I_n}-\mathbf{Z})^{\mathbf{j}}][(\mathbf{I_n}+\mathbf{Z})^{\mathbf{j}}+(\mathbf{I_n}-\mathbf{Z})^{\mathbf{j}}]^{-1} \tag{8}$$

where $\mathbf{Z}\in\mathbf{R}^{n\times n}$ and $\mathbf{I_n}$ is the identity matrix of order n. A fast and stable algorithm for computing the matrix sign function (8) can be found in [18,22].

3. Design of optimal sliding mode controller

We now consider the optimal sliding mode controller design for the state-space model which is both controllable and observable as

$$\dot{x}_c(t)=\mathbf{A}x_c(t)+\mathbf{B}u_c(t)$$
$$y(t)=\mathbf{C}x_c(t), \tag{9}$$

where $\mathbf{A}\in\mathbf{R}^{n\times n}$, $\mathbf{B}\in\mathbf{R}^{n\times m}$, $\mathbf{C}\in\mathbf{R}^{m\times n}$, $x_c(t)\in\mathbf{R}^n$, $u_c(t)\in\mathbf{R}^m$ and $y(t)\in\mathbf{R}^m$. The initial condition is $x_c(0)=\alpha$. The proposed sliding mode controller can be expressed as

$$u_c(t)=-\mathbf{K}_c x_c(t)+\mathbf{E}_c sign(S(t)) \tag{10}$$

where $\mathbf{K}_c\in\mathbf{R}^{m\times n}$ and $\mathbf{E}_c\in\mathbf{R}^{m\times m}$ are control gains calculated by the optimal control law. $S(t)\in\mathbf{R}^m$ is defined as the sliding surface to be