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Further results on global state feedback stabilization of nonlinear high-order feedforward systems

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A R T I C L E I N F O

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A B S T R A C T

In this paper, by introducing a combined method of sign function, homogeneous domination and adding a power integrator, and overcoming several troublesome obstacles in the design and analysis, the problem of state feedback control for a class of nonlinear high-order feedforward systems with the nonlinearity's order being relaxed to an interval rather than a fixed point is solved.

1. Introduction

With the development of control problem for nonlinear systems, e.g., [1–9], in recent years, more attention has been paid on the stabilization of nonlinear feedforward systems as follows:

$$\dot{\eta}_i(t)=\eta_{i+1}^{p_i}(t)+f_i(t,\eta_{i+2}(t),\ldots,\eta_{n+1}(t)),\quad i=1,\ldots,n-1,$$
$$\dot{\eta}_n(t)=v^{p_n}(t), \tag{1}$$

where $\eta_{n+1}(t)=0$, $\eta(t)=[\eta_1(t),\ldots,\eta_n(t)]^\top\in R^n$ and $v(t)\in R$ are the system state and control input, respectively. For $i=1,\ldots,n$, $p_i\in R_{odd}^{\geq 1}\triangleq\{p/q\in R^+:\ p$ and q are odd integers, $p\geq q\}$, $f_i:R^+\times R^{n-i}\to R$ is an unknown continuous function with $f_i(t,0)=0$. System (1) is called as nonlinear high-order system if there exists at least one $p_i>1$.

For $p_i=1$, there are some fruitful results, see [10–17] and the references therein. While for $p_i\geq 1$, due to some intrinsic features of high-order systems, e.g., the Jacobian linearization is neither controllable nor feedback linearizable, there are very few results achieved for feedforward system (1). In [18], the low gain homogeneous domination method is used to achieve the output feedback stabilization for a chain of odd power integrators coupled with nonlinear high-order functions. In [19,20], a state feedback controller is designed for nonlinear high-order feedforward systems with input delay. Zhang et al. [21] investigates the problem of global strong feedback stabilization for nonlinear high-order feedforward time-delay systems. The problem of global output feedback control for a class of nonlinear high-order feedforward systems with input delay is studied in [22], their assumptions can be summarized as the form:

$$|f_i(\cdot)|\leq M\sum_{j=i+2}^{n+1}|\eta_j(t)|^{l_{i,j}} \tag{2}$$

with order $l_{i,j}=p_i\ldots p_{j-1}$ being a fixed number. Immediately, a very interesting problem is asked:

Is it possible to relax the order $l_{i,j}$ to be an interval but not a fixed number? Under the weaker condition, can a stabilizing feedback controller be designed?

In this paper, by introducing a combined method of sign function, homogeneous domination and adding a power integrator, and overcoming several troublesome obstacles in the design and analysis procedure (see Remarks 1–3), we focus on solving the above problem.

This paper is organized as follows: Section 2 gives some useful preliminaries. Sections 3 and 4 provide the design and analysis of controller respectively, following a simulation example in Section 5. Section 6 concludes this paper.

2. Mathematical preliminaries

The following notations and lemmas are to be used throughout the paper.

Notations: R^+ stands for the set of all the nonnegative real numbers. For any vector $x=[x_1,\ldots,x_n]^\top\in R^n$, denote $\bar{x}_i\triangleq[x_1,\ldots,x_i]^\top\in R^i$, $i=1,\ldots,n-1$, $\|x\|\triangleq\sqrt{\sum_{i=1}^n x_i^2}$. A sign function

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sgn(x) is defined as sgn(x) = 1 if $x>0$, sgn(x) = 0 if x=0, and sgn(x) = −1 if $x<0$. The argument of function (or functional) $f(x(t))$ is denoted by $f(x)$, $f(\cdot)$, or f.

Lemma 1 (Qian and Lin [23]). *For $x,y\in R$, $p\ge 1$ is a constant, then $|x+y|^p\le 2^{p-1}|x^p+y^p|$, $(|x|+|y|)^{1/p}\le |x|^{1/p}+|y|^{1/p}$. If $p\in R^{\ge 1}_{odd}$, then $|x-y|^p\le 2^{p-1}|x^p-y^p|$, $|x^{1/p}-y^{1/p}|\le 2^{1-1/p}|x-y|^{1/p}$.*

Lemma 2 (Sun et al. [24]). *If $p=b_1/b_0\in R^{\ge 1}_{odd}$, $b_1\ge b_0\ge 1$, then $|x^p-y^p|\le 2^{1-1/b_0}|\mathrm{sgn}(x)|x|^{b_1}-\mathrm{sgn}(y)|y|^{b_1}|^{1/b_0}$.*

Lemma 3 (Mitrinović [25]). *For $x,y\in R$, then $xy\le \gamma|x|^p+((p\gamma)^{-q/p}/q)|y|^q$, where $p>1$, $q>1$ and $1/p+1/q=1$, γ is any positive constant.*

Lemma 4 (Sun and Liu [26]). *For the continuous function $f:[a,b]\to R(a\le b)$, if it is monotonically increasing and satisfies $f(a)=0$, then $|\int_a^b f(x)\,dx|\le |f(b)||b-a|$.*

Lemma 5 (Sun et al. [24]). *$f(x)=\mathrm{sgn}(x)|x|^a$ is continuously differentiable and satisfies $\dot f(x)=a|x|^{a-1}$, where $a\ge 1$, $x\in R$.*

Lemma 6 (Khalil [4]). *Let $V:R^n\to R$ be a continuous positive definite function. Then there exist class $\mathcal{K}$ functions γ_1 and γ_2 defined on $[0,+\infty)$, such that $\gamma_1(\|x\|)\le V(x)\le\gamma_2(\|x\|)$ for all $x\in R^n$. Moreover, if $V(x)$ is radially unbounded, then γ_1 and γ_2 can be chosen to class $\mathcal{K}_\infty$.*

Lemma 7 (Krstić et al. [1]). *For the nonautonomous system $\dot x=f(x,t)$, let x=0 be an equilibrium point of system and $V:R^n\times R^+\to R^+$ be a continuously differentiable function such that $\gamma_1(\|x\|)\le V(x,t)\le\gamma_2(\|x\|)$ and $\partial V/\partial t+(\partial V/\partial x)f(x,t)\le -\gamma_3(\|x\|)$ hold for any $t\ge 0$, $x\in R^n$, where γ_1 and γ_2 are class $\mathcal{K}_\infty$ functions, and γ_3 is a class $\mathcal{K}$ function on R^+. Then the equilibrium x=0 is globally uniformly asymptotically stable.*

3. State feedback controller

3.1. Problem formulation

Throughout this paper, we assume that f_i satisfies the following condition.

Assumption 1. For each $i=1,\ldots,n-1$, there exist constants $-1/\sum_{l=1}^n p_1\ldots p_{l-1}<\omega\le 0$ and $M>0$ such that

$$|f_i(\cdot)|\le M\sum_{j=i+2}^{n+1}|\eta_j(t)|^{(r_i+\omega)/r_j}, \tag{3}$$

where

$$r_1=1,\quad r_{i+1}=\frac{r_i+\omega}{p_i},\ i=1,\ldots,n. \tag{4}$$

Remark 1. Obviously, system (1) satisfying Assumption 1 is a nonlinear feedforward system.

Next, we focus on discussing the significance of Assumption 1. By $-1/\sum_{l=1}^n p_1\ldots p_{l-1}<\omega\le 0$ and (4), it is easy to see that the order in the growth condition is $l_{i,j}=(r_i+\omega)/r_j\in[p_i\ldots p_{j-1},A)$, where $A=p_i\ldots p_{j-1}(\sum_{l=1}^n p_1\ldots p_{l-1}-\sum_{l=1}^i p_1\ldots p_{l-1})/\sum_{l=1}^n p_1\ldots p_{l-1}-\sum_{l=1}^{j-1}p_1\ldots p_{l-1}$. It is obvious that Assumption 1 is much weaker than that in [18–22] in which $l_{i,j}=p_i\ldots p_{j-1}$. Fig. 1 clearly highlights the significance of Assumption 1.

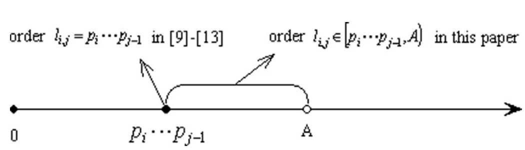

Fig. 1. The value range of the order of nonlinearity.

The purpose of this paper is to design a state feedback controller for system (1) under Assumption 1 such that the closed-loop system is globally asymptotically stable.

3.2. Design of state feedback controller

Introduce the following scaling transformation:

$$x_i=\Gamma^{-d_i}\eta_i,\quad u=\Gamma^{-d_{n+1}}v,\quad i=1,\ldots,n, \tag{5}$$

where $0<\Gamma<1$ is to be designed later, and $d_1=0$, $d_i=(d_{i-1}+1)/p_{i-1}$, $i=1,\ldots,n+1$. Then system (1) can be rewritten as

$$\dot x_i=\Gamma x_{i+1}^{p_i}+\tilde f_i,\quad i=1,\ldots,n-1,$$
$$\dot x_n=\Gamma u^{p_n}, \tag{6}$$

where $\tilde f_i=\Gamma^{-d_i}f_i$ and

$$|\tilde f_i|\le M\Gamma^{1+\nu}\sum_{j=i+2}^{n+1}|x_j|^{(r_i+\omega)/r_j},$$

$$\nu=\min_{i=1,\ldots,n-1,\,j=i+2,\ldots,n+1}\left\{\frac{d_j(r_i+\omega)}{r_j}-d_i-1\right\}$$
$$=\min_{i=1,\ldots,n-1\,j=i+2,\ldots,n+1}\left\{\frac{\omega d_j+\dfrac{1+p_i+p_i\ldots p_{j-2}}{p_1\ldots p_{j-1}}}{\omega d_j+\dfrac{1}{p_1\ldots p_{j-1}}}-1\right\}$$
$$>0. \tag{7}$$

For simplicity, in the following deduction, for any $a\in R^+$ and $x\in R$, we use $[\cdot]$ to denote $[x]^a\triangleq\mathrm{sgn}(x)|x|^a$ rather than a bracket.

For system (6), we have the following proposition.

Proposition 1. *For system (6), there exist a series of constants $\beta_1,\ldots,\beta_n$, a continuous state feedback controller $u(t)$, and a continuously differentiable, positive definite and radially unbounded Lyapunov function $V(\cdot)$, such that*

$$\dot V\le -\Gamma\sum_{j=1}^n a_{n,j}z_j^2+\sum_{j=1}^n\frac{\partial V}{\partial x_j}\tilde f_j, \tag{8}$$

where $a_{n,1},\ldots,a_{n,n}$ are positive constants, and

$$z_j=[x_j]^{1/r_j}-[\alpha_{j-1}]^{1/r_j},$$
$$\alpha_{j-1}=-\beta_{j-1}^{r_j}[z_{j-1}]^{r_j},\quad j=1,\ldots,n. \tag{9}$$

Proof. At step 1, we choose the first Lyapunov function $V_1=l_1\int_0^{x_1}[s]^{2-r_2p_1}ds$ with $l_1>0$ being a constant, then $\dot V_1=\Gamma l_1[x_1]^{2-r_2p_1}(x_2^{p_1}-\alpha_1^{p_1})+\Gamma l_1[x_1]^{2-r_2p_1}\alpha_1^{p_1}+(\partial V_1/\partial x_1)\tilde f_1$. Choosing $\beta_1=(a_{1,1}/l_1)^{1/r_2p_1}$ with $a_{1,1}$ being a positive constant, and using (9), one has $\dot V_1\le -\Gamma a_{1,1}z_1^2+\Gamma l_1[z_1]^{2-r_2p_1}(x_2^{p_1}-\alpha_1^{p_1})+(\partial V_1/\partial x_1)\tilde f_1$. □

Step k (k=2, …, n): At step $k-1$, suppose that there are positive constants β_1, …, β_{k-1} and a Lyapunov function $V_{k-1}(\bar z_{k-1})$ such that

$$\dot V_{k-1}\le -\Gamma\sum_{j=1}^{k-1}a_{k-1,j}z_j^2+\Gamma l_{k-1}[z_{k-1}]^{2-r_{k+1}p_k}(x_k^{p_{k-1}}-\alpha_{k-1}^{p_{k-1}})$$
$$+\sum_{j=1}^{k-1}\frac{\partial V_{k-1}}{\partial x_j}\tilde f_j, \tag{10}$$

with $a_{k-1,1}$, …, $a_{k-1,k-1}$, l_{k-1} being some positive constants. Next, we show that (10) still holds for step k. Choose $V_k=V_{k-1}+l_kW_k$ with $l_k>0$ being a constant, where

$$W_k=\int_{\alpha_{k-1}}^{x_k}[[s]^{1/r_k}-[\alpha_{k-1}]^{1/r_k}]^{2-r_{k+1}p_k}ds. \tag{11}$$

From (4) and $-1/\sum_{l=1}^n p_1\ldots p_{l-1}<\omega\le 0$, it is not hard to know that

$$r_k\ge 1,\quad 2-r_{k+1}p_k\ge 1,\quad k=1,\ldots,n, \tag{12}$$