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Research Article

Asynchronous control of discrete-time impulsive switched systems with mode-dependent average dwell time

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ABSTRACT

This paper mainly studies the asynchronous control problem for a class of discrete-time impulsive switched systems, where "asynchronous" means the switching of the controllers has a lag to the switching of system modes. By using multiple Lyapunov functions (MLFs), the much looser asymptotic stability result of closed-loop systems is derived with a mode-dependent average dwell time (MDADT) technique. Based on the stability result obtained, the problem of asynchronous control is solved under a proper switching law. Moreover, the stability and stabilization results are formulated in form of matrix inequalities that are numerically feasible. Finally, an illustrative numerical example is presented to show the effectiveness of the obtained stability results.

1. Introduction

Switched systems, which consist of a finite number of distinct active subsystems and a switching rule that orchestrates switching between these subsystems, form an important class of hybrid systems. The motivation for studying switched systems comes from the fact that such systems have a lot of applications in the control of systems. Examples include robotics, power electronics, aircraft and air traffic control, and many other fields.

The stability problem, caused by diverse switching, is a major concern in the area of switched systems [1–6,17,18]. So far, two stability issues, the stability under arbitrary switching and the one under constrained switching, have been studied. The main method to solve the former issue is to construct a common Lyapunov function (CLF). As for the issue of constrained switching, an improved method is to use the multiple Lyapunov functions (MLFs) [19,20]. As a class of typical constrained switching signals, many researchers use the average dwell time (ADT) approach to design the switching law. The ADT switching means that the number of switches in a finite time interval is bounded and the average time between consecutive switching is not less than a constant. This method has been widely used to investigate the stability and stabilization problems of constrained switched systems [7–9]. Recently, a new method called mode-dependent average dwell time (MDADT) is proposed in [10]. The new method has been proved to be more applicable than the ADT.

Usually, due to abrupt changes at certain instants during the dynamical process, it inevitably exists impulsive dynamical behaviors. Switched systems with this behavior can be modeled as impulsive switched systems. Therefore, it is important and, in fact, necessary to study impulsive switched systems. Some results can be seen in [11–13]. For example in [11], the author investigated the exponential stability problems of nonlinear impulsive switched systems. The authors in [13] studied the stability and stabilization problems of impulsive switched systems with time-delay.

In practice, when the systems are switching between the subsystems, the matched controllers cannot operate immediately. It inevitably takes some time to identify the system modes and apply the matched controllers. Such a behavior is called asynchronous behavior, and it usually results in unsatisfactory performance and even instability. However, in the past numerous studies, a common assumption is that the controllers are switched synchronously with the switching of system modes, which is quite ideal. So it is necessary to consider this case when stabilizing the switched systems. Recently, the research of asynchronous stabilization and control has become a hot topic, and some related results can be seen in [8,14].

Up to now, most works have been done in connection with impulsive switched systems or asynchronous switched systems.

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To the best of our knowledge, there is no result considering both the impulsive effects and asynchronous behaviors, which inspires us for this study. In this paper, by using MLFs, the much looser asymptotic stability result of closed-loop systems is derived with the MDADT technique. Based on the stability result obtained, a special algorithm is derived to solve the asynchronous stabilization problem.

The remainder of the paper is organized as follows. System descriptions and necessary lemmas are presented in Section 2. In Section 3, stability of closed-loop systems is studied. In Section 4, under a proper switching law, the asynchronous control problem is solved for the impulsive switched systems. In Section 5, numerical examples are presented to testify effectiveness of the obtained results. Finally, a conclusion is given in Section 6.

Notations 1. The notation used in this paper is fairly standard. N and N^+ denote the set of the natural numbers and the set of positive integers respectively. The superscript 'T' stands for matrix transpose. I represents the identity matrix in the block matrix. R^n denotes the n-dimensional Euclidean space. The notation $\|\ \|$ refers to the Euclidean vector norm. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a function $\alpha: [0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\alpha(0)=0$. Also, a function $\beta:[0,\infty)\times[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot,k)$ is of class $\mathcal{K}$ for each fixed $k\ge 0$ and $\beta(s,k)$ decreases to 0 as $k\to\infty$ for each fixed $s\ge 0$. We use $P>0(\ge,<,\le)$ to denote a positive define (semi-positive define, negative define, semi-negative define) matrix P. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix. If not explicitly stated, matrices are assumed to have compatible dimensions.

2. System descriptions and preliminaries

Consider the following class of nonlinear impulsive switched systems:

$$\begin{cases} x(k+1)=A_{\sigma(k_i)}x(k)+B_{\sigma(k_i)}u(k), & k\in(k_i,k_{i+1}] \\ \Delta x(k)=D_{\sigma(k_i)}x(k)+f(k,x(k)), & k=k_i \\ x(k_0^+)=x_0 \end{cases} \quad (1)$$

where $x(k)\in R^n$ is the state, $u(k)\in R^m$ is the control input. $A_{\sigma(k_i)}\in R^{n\times n}$, $B_{\sigma(k_i)}\in R^{n\times m}$, $D_{\sigma(k_i)}\in R^{n\times n}$ are the known matrices. $f(k,x(k))$ is a nonlinear function and $f(k,0)\equiv 0$ for all $k\in[k_0,\infty)$. $\Delta x(k_i)=x(k_i^+)-x(k_i^-)=x(k_i^+)-x(k_i)$, with $x(k_i^-)=x(k_i)=A_{\sigma(k_{i-1})}x(k_i-1)+B_{\sigma(k_{i-1})}u(k_i-1)$. So, we can get $x(k_i^+)=(I+D_{\sigma(k_{i-1})})x(k_i)+f(k_i,x(k_i))$, which means that the state $x(k)$ changes from $x(k_i^-)$ to $x(k_i^+)$ at the switching instant because of the impulsive effects. We define $\sigma(k)$ as a switching signal, which is piecewise constant function of time and takes its values in the finite set $S=\{1,\dots,M\}$, M is the number of subsystems. For a switching sequence $0<k_0<k_1<\cdots k_i<k_{i+1}<\cdots$, $\sigma(k)$, when $k\in(k_i,k_{i+1}]$, we say that the σth(k_i) subsystem is activated, and for $\sigma(k_i)$, $\sigma(k_i)\to\{1,2,\dots,M\}$.

We now consider a class of switched linear feedback controllers $u(k)=K_{\sigma(k_i)}x(k)$, where $K_{\sigma(k_i)}\in R^{m\times n}$ is a constant matrix. We call $u(k)$ as the matched controller. But in practice, it inevitably takes some time to identify the system mode and apply the matched controller, in other words, there is some time that the subsystem is controlled by the unmatched controller. Without loss of generality, we define $\hat{u}(k)=K_{\sigma(k_{i-1})}x(k)$ as the unmatched controllers, where $K_{\sigma(k_{i-1})}\in R^{n\times m}$ is a constant matrix. Then, we obtain the following closed-loop system:

$$\begin{cases} x(k+1)=(A_{\sigma(k_i)}+B_{\sigma(k_i)}K_{\sigma(k_{i-1})})x(k), & k\in(k_i,\overline{k}_i], \\ x(k+1)=(A_{\sigma(k_i)}+B_{\sigma(k_i)}K_{\sigma(k_i)})x(k), & k\in(\overline{k}_i,k_{i+1}], \\ \Delta x(k)=D_{\sigma(k_i)}x(k)+f(k,x(k)), & k=k_i, \\ x(k_0^+)=x_0 \end{cases} \quad (2)$$

where $i\ge 1$, and the notation $\overline{k}_i$, $k_i<\overline{k}_i\le k_{i+1}$ represents the starting-operating instant of matched controller. If $i=0$, the closed-loop system is

$$x(k+1)=(A_{\sigma(k_0)}+B_{\sigma(k_0)}K_{\sigma(k_0)})x(k), \quad k\in[k_0,k_1] \quad (3)$$

which means that there are no impulsive and asynchronous effects at the initial instant.

To obtain the desired results, we now introduce the following assumptions and definitions which will be used in the sequel.

Assumption 1. In order to obtain the desired results, we let the unknown function $f(k,x(k))$ satisfy the following inequalities:

$$\|f(k,x(k))\|\le\eta\|x(k)\|$$

for all $k\in[k_0,\infty)$, where η is a positive constant.

Definition 1 (*Liberzon* [1]). The switched system (1) with $u(k)\equiv 0$ is globally uniformly asymptotically stable (GUAS) if there exists a class $\mathcal{KL}$ function β such that for all switching signals σ and all initial condition $x(k_0)$, the solutions of (1) satisfy the inequality

$$\|x(k)\|\le\beta(\|x(k_0)\|,k), \quad \forall k\ge k_0.$$

Definition 2 (*Zhao et al.* [10]). For switching signal $\sigma(k)$ and each $K\ge k\ge 0$. Let $N_p(K,k)$ be the switching numbers that the pth subsystem is activated over the interval $[k,K]$ and $T_p(K,k)$ denote the total running time of the pth over the interval $[k,K]$. We say that $\sigma(k)$ has a mode-dependent average dwell time (MDADT) τ_p if there exist positive numbers N_{0p} (we call N_{0p} the mode-dependent chatter bounds here) and τ_p such that

$$N_p(K,k)\le N_{0p}+\frac{T_p(K,k)}{\tau_p}, \quad \forall K\ge k\ge 0,\ \forall p\in S.$$

Remark 1. Definition 2 constructs a novel set of switching signals with a MDADT property, it means that if there exist positive numbers τ_p, $p\in S$ such that a switching signal has the MDADT property, we only require the average time among the intervals associated with the pth subsystem is larger than τ_p (note that the intervals here are not adjacent).

In order to complete the proof, the following lemmas are also needed.

Lemma 1 (*Xu and Teo* [11]). *Let $P\in R^{n\times n}$ be a given symmetric positive definite matrix and let $Q\in R^{n\times n}$ be a given symmetric matrix. Then*

$$\lambda_{\min}(P^{-1}Q)\Omega(k)\le x(k)^TQx(k)\le\lambda_{\max}(P^{-1}Q)\Omega(k)$$

for all $x(k)\in R^n$, where $\Omega(k)=x(k)^TPx(k)$, while $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the largest and the smallest eigenvalues of the matrix inside the brackets.

Lemma 2 (*Shi et al.* [15]). *Given matrices M, E and F with compatible dimensions and F satisfying $F^TF\le I$, then the following inequalities hold for any $\varepsilon>0$:*

$$MFE+E^TF^TM^T\le\varepsilon MM^T+\varepsilon^{-1}E^TE.$$