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Optimal temperature control for quality of perishable foods

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ABSTRACT

An optimal control model on temperature control for quality of perishable foods is presented in this paper, in which the quality deteriorates according to the first-order reaction and the temperature is defined as a control variable. The objective functional includes the loss induced by quality deterioration and the cost of temperature control. To minimise the total cost, the optimal temperature is obtained by solving the optimal control problem with Pontryagin's maximum principle. It is strictly proved that the isotherm condition of storage is optimal, which can be implemented conveniently in practice and improve the economic benefits of enterprises. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

In recent years, more and more attention has been paid to the inventory systems of perishable foods in the literatures due to its strategic importance [1,2]. However, since the quality of the perishable foods deteriorates with time, the management of perishable foods is far from satisfactory and the loss of perishable foods can be as high as 15% due to damage and spoilage [3]. Inappropriate quality control or excessive inventories are the main reason for the loss in a supply chain of perishable foods.

Lately food safety has acquired a great importance due to the globalisation and free trade in food [4,5]. Consumers have increasingly concerned about the quality and safety of perishable foods especially the highly perishable foodstuffs, such as fresh meat, fresh fish, dairy products, etc. The quality of perishable foods is one of the most essential characteristics to be considered throughout supply chain. Generally speaking, perishable foods have a limited lifetime which is a function of product characteristics, storage conditions under which the product is maintained, and time [6]. For firms, the perishable foods must be sold to consumers within the shelf life so as to ensure quality and safety while maximising profit. It is of vital importance to maintain the quality of perishable foods which depends on the environmental conditions of storage and transportation [7]. Perishable foods quality can be considered as a dynamic state that decreases continuously with time until the time point when it is unsuitable for sale or consumption because of its nature [8]. In the existing literatures, many models have been presented to model the quality evolution of perishables [9,10]. However, for the perishable foods, it is difficult to estimate its quality because of the range of quality attributes, dynamics of product characteristics and storage conditions. The quality deterioration of perishable foods in the storage is affected by storage time, storage temperature, and other various factors such as activation energy, gas constant, etc., among which the temperature plays a key role in maintaining the quality. In general, most perishable foods are temperature sensitive and the temperature is the main environmental condition resulting in quality degradation [11]. Therefore, temperature control is essential to ensure the quality and safety of the perishable foods.

Nahmias [12] and Raafat [13] studied the basic models of production and inventory systems with deteriorating items. In these articles, products perishability was considered to create uncertainty for buyers with respect to product quality, safety and supply's reliability. Goyal [14] presented a comprehensive review of the literatures of the deteriorating inventory models. However, the temperature is seldom incorporated by the existing inventory-theory-based modelling method due to the technical difficulties. Benítez et al. [15] pointed out that it was of critical importance to control temperature during refrigerated storage. With modern technologies widely adopted, such as ratio frequency identification technology (RFID), time temperature indicator (TTI), one can automatically capture product information regarding product identity, properties and related data (e.g., temperature, humidity, etc.) in real time [8,16]. Such transparency generates the possibility that the perishable foods quality can be dynamically predicted

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based on the environmental conditions during storage. Thus, it is possible to maintain the quality of perishable foods through controlling environmental conditions especially the temperature during storage.

Model-based optimisation methods have been developed during the last decades and these methods have usefulness and great potential for improving food processing [17]. Rong et al. [7] presented a mixed-integer linear programming model for production and distribution planning in a foods supply chain, where the quality of perishable foods was integrated in decision-making. To optimise temperature control, different levels of possible temperature are given beforehand in the storage. It enables to have model select the optimal storage temperature among these temperature levels. Zhang et al. [18] developed the kinetic models to predict freshness changes of grass carp at different temperatures during storage in order to study quality changes in cold chain circulation. Nevertheless, to the best of our knowledge, up to now there is no criterion to guide how to set the optimal temperature in the storage under different system parameters.

This paper focuses on how to determine the optimal temperature in the storage. We consider the impact of temperature on the quality of perishable foods, and construct the optimal control model of temperature control which minimises the total cost including the loss induced by spoilage and the cost of controlling temperature. The optimal temperature is found by solving the optimal control problem with the Pontryagin's maximum principle. Different from the method of finding the optimal temperature among different pre-set levels of possible temperature, the optimal control method can provide a guideline for setting the global optimal temperature in the storage with different system parameters and can be adopted to design and/or operate (control) food processes in an optimal way resulting in the minimum cost.

The rest of this paper is organised as follows: In Section 2, the model of optimal temperature control with the objective of minimising the total cost is presented. In Section 3, the optimal temperature is obtained by solving the optimal control problem with Pontryagin's maximum principle. In Section 4, a numerical example is given to illustrate the effectiveness of the proposed method. Finally, Section 5 makes a conclusion.

2. Modelling optimal temperature control

The model under consideration is developed with the following assumptions:

(i) There is only one kind of perishable foods is saved in a storage in a certain period $[0, t_f]$.
(ii) Taking foods into or out of a warehouse is not allowed within the period $[0, t_f]$.

In the following, the dynamic of quality degradation is introduced which plays an important role in modelling the quality of perishable foods. According to Labuza [19], the model of quality deterioration is described as

$$\dot{q}(t) = kq^n(t), \tag{1}$$

where $q(t)$ denotes the quality of product at time of t, k is the deterioration coefficient which depends on the environmental conditions in the storage, and n is the order of the reaction. Denote the initial quality as q_0, i.e., $q(0) = q_0$.

The reaction order n in (1) takes either 0 or 1 (zero-order or first-order reactions) in most situations, resulting in a linear or exponential quality decay respectively. In this paper, we consider the first-order reaction, i.e., $n = 1$.

Based on the Arrhenius equation which is a formula for the temperature dependence of a chemical reaction [20,21], the general form of the deterioration coefficient k is

$$k = k_0 \exp\left(\frac{-E_0}{RT(t)}\right), \tag{2}$$

where k_0 is the pre-exponential constant, E_0 is the activation energy for the reaction, R is the ideal gas constant. $T(t)$ described by the absolute temperature is the storage temperature that can be adjusted continuously at any time, i.e., a function of time t. Denote the controllable temperature interval as $[\underline{T}, T_0]$ with the lowest adjustable temperature $\underline{T}$ and the highest adjustable temperature T_0. In practice, the maximum temperature T_0 is generally set to be the natural temperature. According to (1) and (2), it is possible to identify the quality level of the perishable foods with the given initial quality, storage time and deterioration coefficient.

In the storage, different temperatures set for the preservation of perishable foods imply different costs [21]. Generally speaking, a lower temperature on one hand means a better storage condition which can slow down the deterioration of quality as shown in (1) and (2), and on the other hand results in a higher cost of implementation. Lowing cost of temperature control by increasing temperature may boost quality degradation, thus increasing the loss in value.

More specifically, according to the results obtained in Dorato [22], the cost of controlling temperature can be expressed as

$$P_c(T) = \int_0^{t_f} (Q_{aux} + \rho(T(t) - T_0)^4)\, \mathrm{d}t, \tag{3}$$

where Q_{aux} represents consumption of auxiliary energy, ρ is a positive cost coefficient of adjusting temperature, and $\rho(T(t) - T_0)^4$ represents a penalty for deviation from the desired temperature, i.e., the natural temperature. The bigger the deviation between T and T_0 is, the higher the cost of controlling temperature is.

The form of quartic function of the deviation between ambient temperature and desired temperature has been adopted in the existing literature, such as Dorato [22] and Somasundaram et al. [23]. According to Somasundaram et al. [23], the cost of controlling temperature is a quartic function of the control accuracy. From a technical point of view, quartic function is convex function which can be treated analytically.

The perishable foods are subjected to quality degradation over time even when they are properly stored. Thus, there is a loss induced by the deterioration of quality. At the terminal time t_f, the quality is $q(t_f)$ which induces a loss represented by $\Phi(q(t_f))$. Indeed, $\Phi(q(t_f))$ can be regarded as a penalty of spoilage during the period $[0, t_f]$. The more the spoilage is, the higher the penalty is. Thus, it is reasonable to assume that

$$\frac{\mathrm{d}\Phi}{\mathrm{d}q} > 0, \quad \frac{\mathrm{d}^2\Phi}{\mathrm{d}q^2} > 0. \tag{4}$$

Consequently, the total cost including the loss caused by spoilage and the cost of temperature control is described by

$$J = \Phi(q(t_f)) + \int_0^{t_f} (Q_{aux} + \rho(T(t) - T_0)^4)\, \mathrm{d}t. \tag{5}$$

The quality $q(t)$ and the temperature $T(t)$ are regarded as the state variable and the control variable of the dynamic system respectively. Thus, an optimal control model to minimise the total cost is given as

$$\begin{cases} \min\limits_{T(\cdot)} \quad J = \Phi(q(t_f)) + \int_0^{t_f} (Q_{aux} + \rho(T(t) - T_0)^4)\, \mathrm{d}t \\ \text{s.t.} \quad \dot{q}(t) = k_0 \exp\left(\dfrac{-E_0}{RT(t)}\right) q(t), \quad q(0) = q_0, \\ \qquad \underline{T} \le T(t) \le T_0. \end{cases} \tag{6}$$