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Research Article

# Tuning rules for a quick start up in Dynamic Matrix Control

Clemente Manzanera Reverter [a,*], Julio Ibarrola [b], José-Manuel Cano-Izquierdo [b]

[a] Polytechnic University of Cartagena, C/Príncipe de Asturias 20 4°Derecha, C.P. 30204 Cartagena, Murcia, Spain
[b] Polytechnic University of Cartagena, Universidad Politécnica de Cartagena, Departamento de Sistemas y Automática, Campus Muralla de Mar Calle Doctor Fleming, s/n, C.P. 30202 Cartagena, Murcia, Spain

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ABSTRACT

This paper pretends to offer design rules for the parameters adjustment of the Dynamic Matrix Control (DMC) to allow an easier starting up. The effect on the time response of each algorithm parameter that can be tuned by the user is studied in an unconstrained system. To this aim, the position of the closed loop poles of the equivalent system is calculated. To simplify the study and to obtain more direct conclusions the number of poles will be limited using a First Order Plus Death Time simplification of the real plant. Design rules proposed in this study are tested in some simulated benchmarks and in a real plant.



## 1. Introduction

Dynamic Matrix Control (DMC) has become a popular Model Predictive Control (MPC) method since it was first introduced by Cutler and Ramaker [12] in the last 70s. It is one of the most used algorithms in industry, but a method for setting its parameters is still being investigated.

There are some mathematical techniques to tune these parameters. A well known algorithm is the one presented by Shridhar and Cooper [1] who introduced a method to calculate the weighting factor minimizing the condition number of the system matrix. For its calculation the system is approximated by a First Order Plus Dead Time (FOPDT) system. This method is one of the most extended and has been studied by several researchers [16,14]. Another example is the algorithm presented by Trierweiler and Farina [2] that uses a Robustness Performance Number (RPN) which indicates how difficult is for a system to reach the required performances with robustness. This method gives directives to calculate the prediction horizon, the control horizon and the sample time. It calculates the system's weighting matrix based on the RPN. This method modifies the normal cost function when it factorizes the system matrix. Han et al. [3] propose a minimization–maximization algorithm over a performance index. Garriga and Soroush propose tuning via eigenvalue placement [11].

Some works face a more practical approach using thumb rules given by the experience obtained from simulations and real controllers. This is a usual approach in industry. The work from Iglesias et al. [4] is an example of this. They present a formula obtained by correlation with data from several simulations. Bagheri and Khaki-Sedigh [17] propose an analysis of variance. Wojsznis et al. present the use of heuristic methods [18]. In this category auto-tuning methods could be included [8].

Previous works agree on the effect of control horizon and weighting factor but a consensus about what parameter, prediction horizon and weighting factor should be taken as key parameter is not found. Some authors (as Shridhar and Cooper [1]) state that the weighting factor is the key parameter to DMC tuning. But others (as Rossiter [5]) doubt this parameter and defend that the prediction horizon is the factor DMC users should focus in.

Following this goal and trying to make the tuning task easier, this paper pretends to obtain some design rules analysing the effect of changes of DMC parameters on the system closed loop poles (a similar approach to the one used in [11]) in a SISO and unconstrained problem. Time response simulations will be done to evidence the obtained results. These rules will allow users to easily obtain a first set of suitable parameters and help them to predict the effect of a parameter's change on the systems performance. To compute the poles of a DMC controlled system, it will be expressed as a Linear Time Invariant (LTI) (this development can be seen in [6]).

This paper is structured as follows: The first section will be an introduction to the DMC formulation and the DMC expressed as an LTI system. This will allow a better understanding of the following

* Corresponding author. Tel.: +34 968 083219; fax: +34 626048569.
*E-mail addresses:* clmanzanera@hotmail.com (C. Manzanera Reverter), juliojose.ibarrola@upct.es (J. Ibarrola), josem.cano@upct.es (J.-M. Cano-Izquierdo).

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section, an analysis of the effect of DMC parameters in closed loop poles and time response from which useful tuning rules will be obtained. The last section will show a validation of the previously mentioned tuning rules by simulation on a benchmark and test on a real system.

## 2. DMC algorithm

As the starting point of this paper is transforming the DMC algorithm in an LTI system, it is mandatory to explain this process. The following paragraphs explain the basis of DMC and how it can be expressed as an LTI system.

### 2.1. DMC formulation

DMC algorithm uses a plant's step response model:

$$y(t)=\sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{1}$$

where $g_i$ are the coefficients of the unit step response, $\Delta u$ is the control increment, $y$ is the system response and $n(t)$ are the disturbances. So predicted values will be (starting predictions from instant $t$)

$$\begin{aligned}\hat{y}(t+k) &= \sum_{i=1}^{\infty} g_i \Delta u(t+k-i)+\hat{n}(t+k)\\ &= \sum_{i=1}^{k} g_i \Delta u(t+k-i)+\sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i)+\hat{n}(t+k)\end{aligned} \tag{2}$$

Considering constant disturbances ($y_m(t)$ being the measured output),

$$\hat{n}(t+k)=\hat{n}(t)=y_m(t)-\hat{y}(t)=y_m(t)-\sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{3}$$

Then Eq. (2) can be written as

$$\begin{aligned}\hat{y}(t+k) &= \sum_{i=1}^{k} g_i \Delta u(t+k-i)+\sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i)+y_m(t)\\ &\quad -\sum_{i=1}^{\infty} g_i \Delta u(t-i)=\sum_{i=1}^{k} g_i \Delta u(t+k-i)+f(t+k)\end{aligned} \tag{4}$$

$f(t+k)$ being the free response, the part of the response not depending on future control actions described as follows:

$$f(t+k)=y_m(t)+\sum_{i=1}^{\infty}(g_{k+i}-g_i)\Delta u(t-i) \tag{5}$$

If the process is asymptotically stable, coefficients of step response, $g_i$, will tend to a constant value after $N$ sample periods, so

$$(g_{k+i}-g_i)\to 0, \quad i>N \tag{6}$$

And Eq. (5) can be simplified to

$$f(t+k)=y_m(t)+\sum_{i=1}^{N}(g_{k+i}-g_i)\Delta u(t-i) \tag{7}$$

Applying the previous equations for a prediction horizon equal to $Pr$ and a control horizon equal to $M$:

$$\hat{y}(t+1/t)=g_1\Delta u(t)+f(t+1) \tag{8}$$

$$\hat{y}(t+2/t)=g_2\Delta u(t)+g_1\Delta u(t+1)+f(t+1) \tag{9}$$

$$\vdots$$

$$\hat{y}(t+Pr/t)=\sum_{i=Pr-M+1}^{Pr} g_i \Delta u(t+Pr-i)+f(t+Pr) \tag{10}$$

Defining the system dynamic matrix as

$$\mathbf{G}=\begin{pmatrix} g_1 & 0 & \dots & 0\\ g_2 & g_1 & \dots & 0\\ g_M & g_{M-1} & \dots & g_1\\ \vdots & \vdots & \dots & \vdots\\ g_{Pr} & g_{Pr-1} & \dots & g_{Pr-M+1}\end{pmatrix} \tag{11}$$

Using matricial formulation, it can be written that

$$\hat{\mathbf{y}}=\mathbf{G}\boldsymbol{\Delta}\mathbf{u}+\mathbf{f} \tag{12}$$

$\hat{\mathbf{y}}$ being a $Pr$-dimensional vector that contains the future system predictions in the prediction horizon, $\boldsymbol{\Delta}\mathbf{u}$ an $M$-dimensional vector that contains the control increments and $\mathbf{f}$ the free response vector. This expression relates the future outputs with the control increments and is used to calculate the necessary action to reach a specific behaviour.

DMC's objective is finding a control increment that minimizes a determined cost function that includes errors and control efforts:

$$J=\sum_{j=1}^{p}(\hat{y}(t+j\,|\,t)-w(t+j))^2+\sum_{j=1}^{m}\lambda(\Delta u(t+j-1))^2 \tag{13}$$

$$\mathbf{J}=\mathbf{e}\mathbf{e}^{\mathbf{T}}+\lambda\boldsymbol{\Delta}\mathbf{u}\boldsymbol{\Delta}\mathbf{u}^{\mathbf{T}} \tag{14}$$

where $\mathbf{e}$ is the errors vector and $\boldsymbol{\Delta}\mathbf{u}$ the control efforts vector. In a problem without constraints, the optimized control efforts vector is obtained solving

$$\frac{\mathbf{dJ}}{\mathbf{d}\boldsymbol{\Delta}\mathbf{u}}=0 \tag{15}$$

the result being

$$\boldsymbol{\Delta}\mathbf{u}=(\mathbf{G}^{\mathbf{T}}\mathbf{G}+\lambda\mathbf{I})^{-1}\mathbf{G}^{\mathbf{T}}(\mathbf{w}-\mathbf{f}) \tag{16}$$

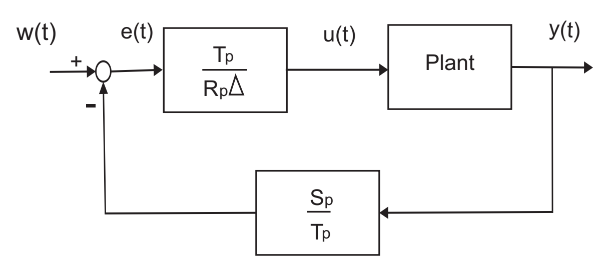


**Fig. 1.** DMC as an LTI system.

**Table 1**
Poles for $T=8$ s.

| $Pr=4$ | $Pr=8$ | $Pr=12$ | $Pr=20$ |
|---|---|---|---|
| −0.5 | −0.6 | −0.63 | −0.63 |
| −0.0728+0.5242i | −0.1402+0.6233i | −0.1747+0.6303i | −0.1887+0.6054i |
| −0.0728−0.5242i | −0.1402−0.6233i | −0.1747−0.6303i | −0.1887−0.6054i |
| 0.7814+0.3091i | 0.6406+0.4251i | 0.5787+0.4197i | 0.5275+0.3929i |
| 0.7814−0.3091i | 0.6406−0.4251i | 0.5787−0.4197i | 0.5275−0.3929i |
| 0.76 | 0.85 | 0.88 | 0.91 |

**Table 2**
Poles for $T=16$ s.

| $Pr=2$ | $Pr=4$ | $Pr=6$ | $Pr=10$ |
|---|---|---|---|
| −0.29 | −0.45 | −0.52 | −0.55 |
| 0.7418+0.2668i | 0.4938+0.5175i | 0.394+0.5588i | 0.3145+0.5327i |
| 0.7418−0.2668i | 0.4938−0.5175i | 0.394−0.5588i | 0.3145−0.5327i |
| 0.49 | 0.77 | 0.81 | 0.85 |