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Simulation and stability analysis of neural network based control scheme for switched linear systems

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ABSTRACT

This paper proposes a new adaptive neural network based control scheme for switched linear systems with parametric uncertainty and external disturbance. A key feature of this scheme is that the prior information of the possible upper bound of the uncertainty is not required. A feedforward neural network is employed to learn this upper bound. The adaptive learning algorithm is derived from Lyapunov stability analysis so that the system response under arbitrary switching laws is guaranteed uniformly ultimately bounded. A comparative simulation study with robust controller given in [Zhang L, Lu Y, Chen Y, Mastorakis NE. Robust uniformly ultimate boundedness control for uncertain switched linear systems. Computers and Mathematics with Applications 2008; 56: 1709–14] is presented.

1. Introduction

A switched system is an important class of hybrid dynamical system which consists of a collection of subsystems and a switching law that decides which of the subsystem is active at each moment. There are many applications of switched systems in control of mechanical systems, process control, power systems, traffic management, and many other practical control fields. The motivation for studying switched systems is from the fact that many practical systems are inherently multi-models. In last two decades, there have been numerous control design methods and stability analysis available in the literature for switched systems [1–7].

A main problem which is always inherent in all dynamical systems is the presence of uncertainties and external disturbances. Even for switched systems this problem has been an active area of research for many years. In [8], a robust $H\infty$ control scheme is proposed for switched linear systems with norm-bounded time-varying uncertainties by using a multiple Lyapunov function method. In [9], an adaptive robust stabilizing controller is designed for a class of uncertain switched linear systems, in which uncertainties satisfying the matching condition and switching among subsystems are determined by using a nominal system. A new class of uncertain switched fuzzy systems is proposed in [10]. In [11], an algorithm for robust adaptive control for parametric-strict output feedback switched systems is developed. This control scheme guarantees system stability for bounded disturbances and parameters without requiring a priori knowledge on such parameters or disturbances. In [12], L2-gain analysis and control synthesis problem is designed for a class of uncertain switched linear systems subject to actuator saturation and external disturbances. The problem of optimal switching for a class of switched systems with parameter uncertainty is proposed in [13]. The conditions for robust stabilization of this class of switched systems with parameter uncertainty are presented based on a multi-Lyapunov function technique and a Linear matrix inequality technique. In [14], a continuous state feedback control scheme for uncertain switched linear systems is proposed in which external disturbance is not considered and the exact knowledge of the possible upper bound of uncertainty is known a priori. However in practical applications the calculation of this bound is difficult and time consuming. Therefor further improvement is needed for this type of control scheme.

Artificial neural network (ANN) can be defined as nonlinear systems consisting of a number of interconnected processing units or neurons. Due to their versatile features such as learning capability, nonlinear mapping and parallel processing, ANN is successfully applicable in real life problems arising in the areas such as dynamics of economy, system control, robotics, etc. The limitations of ANN come from its architecture and the accuracy of the results vary depending on network quality. A European study

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on the Stimulation Initiative for European Neural Applications (SIENA project) indicates ANN is used in 39% of the production or manufacturing sectors, with 35% of the usage related to control, monitoring, modeling and optimization [15]. In [16], the possibility to apply neural network modeling for simulation and prediction of the EU-27 global index of production and domestic output price index behavior is investigated. The most useful property of neural network in control is their ability to approximate arbitrary linear or nonlinear functions through learning. Due to this property neural networks have proven to be a suitable tool for controlling complex nonlinear dynamical systems. The basic idea behind neural network based control is to learn unknown nonlinear dynamics and compensate for uncertainties existing in the dynamic model. There are many NN based control schemes available for a class of switched systems [17–19]. But the case of uncertainty bound estimation using neural network is not found in the literature for switched systems.

In this paper, we propose a new adaptive neural network based control scheme for switched linear systems with parametric uncertainties and external disturbances. The proposed control scheme has the following salient features:

(i) prior information of the upper bound of the uncertainty is not required,
(ii) the feedforward neural network is able to learn the upper bound of uncertainty, and
(iii) uniformly ultimately bounded (UUB) stability of system response and NN weight error are guaranteed under arbitrary switching laws.

This paper is organized as follows. Some basics of stability analysis are given in Section 2. In Section 3, a review of feedforward neural network and a new robust adaptive controller is proposed. A common Lyapunov function approach is used to show that system response and NN weight error are all UUB. A simulation example is provided in Section 4 to illustrate the effectiveness of the proposed control scheme. Section 5 gives conclusions.

2. Preliminaries

Consider a switched linear system represented by a differential equation of the form

$$\dot{x}(t) = A_{\sigma(t)}(\omega)x(t) + B_{\sigma(t)}(\omega)u(t), \quad (1)$$
$$\sigma(t) : R^+ \longrightarrow S = \{1, \dots, N\}$$

where $t \geq 0$, $x = (x_1, x_2, \dots, x_n) \in R^n$ denotes the state vector of the system. $u(t) \in R^m$ is the control input and R^+ denotes the set of nonnegative real numbers. $\sigma(t)$ is a piecewise constant function called a switching law, which indicates the active subsystem at each instant. $A_i(\omega)$, $B_i(\omega)$, $i = 1, \dots, N$ are matrices whose elements are continuous functions of a time-varying vector ω on a compact set $\Omega \subset R^q$. When switched system (1) has parametric uncertainty and external disturbance then it can be written in the following form

$$\dot{x}(t) = (\bar{A}_i + \Delta A_i(\omega))x(t) + (\bar{B}_i + \Delta B_i(\omega))u(t) + d(t), \quad (2)$$

where $d(t)$ is the bounded external disturbance, $\bar{A}_i$, $i = 1, \dots, N$ are commuting Hurwitz matrices and $\Delta A_i(\omega)$, $\Delta B_i(\omega)$ are uncertainty terms which satisfy the following conditions [20,21]

$$\Delta A_i(\omega) = \bar{B}D_i(\omega) \quad (3)$$

$$\Delta B_i(\omega) = \bar{B}E_i(\omega) \quad (4)$$

$$I + \frac{1}{2}(E_i(\omega) + E_i^T(\omega)) \geq \delta I \quad (5)$$

where δ is a positive constant, $D_i : \Omega \longrightarrow R^{m\times n}$ and $E_i : \Omega \longrightarrow R^{m\times m}$ are continuous matrix functions.

Now consider the nominal case with $u(t) = 0$ and $d(t) = 0$

$$\dot{x}(t) = A_{\sigma(t)}(\omega)x(t). \quad (6)$$

Then stability conditions for system (6) are given by the following theorems [22].

Theorem 2.1. *If $\{A_i : i = 1, \dots, N\}$ is a set of commuting Hurwitz matrices, then the switched linear system (6) is globally uniform asymptotic stable for any arbitrary switching sequence between A_i.*

Theorem 2.2. *For a given symmetric positive definite matrix Q, let $P_1, P_2, \dots, P_N > 0$ be the unique symmetric positive definite solutions of the following equations*

$$A_1^T P_1 + P_1 A_1 = -Q, \quad (7)$$

$$A_i^T P_i + P_i A_i = -P_{i-1}, \quad (8)$$

with the condition of Theorem 2.1, then the function $L = x^T P_N x$ is a common Lyapunov function for the switched linear system (6).

Because of the parametric uncertainty and external disturbance in switched linear system (2), we cannot derive the state response $x(t)$ which exactly converges to the equilibrium point. Therefore it is only reasonable to expect that the system response converges into a neighborhood of the equilibrium point and remains within it thereafter, which is the so-called uniformly ultimate boundedness. In [23], the following uniformly ultimate boundedness is proposed for dynamical systems.

Definition 2.1. The solution of a dynamical system is uniformly ultimately bounded (UUB) if there exists a compact set $S \subset R^n$ such that for all $x(t_0) = x_0 \in S$, there exists a $\lambda > 0$ and a number $T(\lambda, x_0)$ such that $\|x(t)\| < \lambda$ for all $t \geq t_0 + T$.

Theorem 2.3. *If there exists a function L with continuous partial derivatives such that for x in a compact set $S \subset R^n$, L is positive definite and $\dot{L} < 0$ for $\|x\| > R$, $R > 0$ such that the ball of radius R is contained in S, then the system is UUB and the norm of the state is bounded to within some neighborhood of R.*

3. Neural network based control design

In this section, our aim is to design a control input $u(t)$ in such a way that the switched linear system (2) is uniformly ultimately bounded (UUB). The following robust control scheme

$$u(t) = \begin{cases} -\dfrac{w}{\|w\|}\rho, & \text{if } \|\omega\| > \epsilon \\ -\dfrac{w}{\epsilon}\rho, & \text{if } \|\omega\| \leq \epsilon \end{cases} \quad (9)$$

is proposed in [14], where $w = \bar{B}^T P_N \rho x$, $\epsilon > 0$ is any constant and ρ denotes the possible upper bound of the parametric uncertainty defined as

$$\rho = \frac{1}{\delta} \max_i \max_{\omega\epsilon\Omega} \|D_i(\omega)\| \, \|x\| \quad (10)$$

which is assumed to be known.

However, there are some potential difficulties associated with the robust controller (9). (i) Implementation requires a precise knowledge of the uncertainty bound, (ii) if an uncertain switched system has many subsystems, the computation of the uncertainty bound will be a complex and time consuming task, and (iii) if a new switched system comes in, the whole procedure has to start again. Therefore, we introduce a new adaptive control approach to avoid the requirement of a prior knowledge of the upper bound ρ in the expression (10) by using a feedforward neural network (FFNN). Mathematically, a two-layer feedforward neural network (Fig. 1) with n input units, m output units and L units in the hidden layer is given as