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Reconstruction of electrical impedance tomography (EIT) images based on the expectation maximum (EM) method

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ABSTRACT

Electrical impedance tomography (EIT) calculates the internal conductivity distribution within a body using electrical contact measurements. The image reconstruction for EIT is an inverse problem, which is both non-linear and ill-posed. The traditional regularization method cannot avoid introducing negative values in the solution. The negativity of the solution produces artifacts in reconstructed images in presence of noise. A statistical method, namely, the expectation maximization (EM) method, is used to solve the inverse problem for EIT in this paper. The mathematical model of EIT is transformed to the non-negatively constrained likelihood minimization problem. The solution is obtained by the gradient projection-reduced Newton (GPRN) iteration method. This paper also discusses the strategies of choosing parameters. Simulation and experimental results indicate that the reconstructed images with higher quality can be obtained by the EM method, compared with the traditional Tikhonov and conjugate gradient (CG) methods, even with non-negative processing.

1. Introduction

Electrical impedance tomography (EIT) has been investigated extensively during the past decades as a visualization and measurement technique. Its aim is to produce images by computing electrical conductivity within the object. Sinusoidal electrical currents are applied to volume using electrodes, and the resulting potentials on the electrodes are measured. EIT has numerous applications in biomedicine, industry and geology. Many potential applications have been developed for both medical and industrial use [1–4].

EIT has several advantages over other tomography techniques, e.g. portability, safety, low cost, non-invasiveness and rapid response. Thus it could provide a novel imaging solution. However, due to the limitation of the number of sensing electrodes and the non-linear property of the field, the imaging reconstruction of EIT is a typical non-linear and ill-posed inverse problem, which is unstable with respect to measurement and modeling errors [5]. Regularization is a good way to solve such a problem. Among the regularization methods, the Tikhonov method has been generally accepted as an important one [6]. However, the traditional regularization methods cannot avoid introducing negative values in the solution, i.e. the gray level of reconstructed image. The negativity of the gray level, which should be positive in real image or conductivity distribution, produces artifacts in reconstructed images in presence of noise. Compared with these methods, statistical techniques can obtain non-negative solution and lower image distortion [7]. Furthermore, statistical models provide a rigorous, effective means with which to deal with measurement error. As a result, tomographic image reconstruction using statistical methods can provide more accurate system models, statistical models, and physical constraints than the conventional method [8].

As a statistical method, the expectation maximization (EM) algorithm is often used to estimate a Poisson model from incomplete data, i.e. data with imperfect values, or with latent variables [9]. Furthermore, the noise level of the measurement system can also be considered as prior information in the EM method. Thus it is robust to measurement noise. The EM method has been widely used for "hard-field" imaging, which is based on the Poisson statistical model, e.g. gamma-ray tomography, X-ray tomography, emission computed tomography (ECT) etc. [10–13]. The basic principle of "hard-field" imaging is to measure the attenuation of the intensity of the radiation described by the Beer–Lambert law [14]. The sensitivity field is not influenced by the distribution of the components in the process being imaged, i.e. the sensor field is not deformed by the process and is equally sensitive to the process parameter in all positions throughout the measurement volume. The sensitivity is also independent of the process component distribution inside the measurement volume. "Hard-filed" sensors are typically nucleonic and optical.

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The sensitivity filed for EIT is non-linear, the sensitivity distribution inside the field depends on the measured media, i.e. it has the property of "soft-field"[15]. As a result, it has low spatial resolution although can be very fast for flow measurement. With "soft-field" sensors, the sensor filed is sensitive to the component parameter distribution inside the measurement volume, in addition to the position of the component, i.e. the measured parameter, in the measurement volume. Thus, the sensor type generates an inhomogeneous sensor field which is changed by the phase distribution and the physical properties of the process being imaged, meaning that the field equipotential is distorted by variation of the electrical properties within the measurement volume. The sensitivity distribution inside the field depends on the parameter distribution.

Under some prior information, the mathematical models of both the "soft-field" and "hard-field" imaging can be united. As a result, the EM method is expected to solve the ill-posed problem for EIT reconstruction. This paper presents the study of the EM method for EIT reconstruction. The selection of parameters for the EM method is also discussed. Both simulation and experimental tests are conducted in order to prove the performance of the EM method. The results are reported and compared with those by using the Tikhonov and CG methods.

2. Typical reconstruction algorithms

In EIT, an array of electrodes (16 electrodes in this paper) is arranged with equispaced in a single plane around the perimeter of the medium and a sinusoidal current are injected through these electrodes. With the adjacent drive pattern, current is applied to an adjacent pair of electrodes and the resultant voltages between the remaining 13 adjacent pairs of electrodes are measured. The three possible measurements involving one or both of the current injecting electrodes are not used. This procedure is repeated 16 times with current injected between successive pairs of adjacent electrodes until all 16 possible pairs of adjacent electrodes have been used to apply the known current [15]. This is shown schematically in Fig. 1. This procedure produces $16\times13=208$ voltage measurements called an EIT data frame.

An estimate for the changes in cross-sectional conductivity distribution of the object is obtained by using the voltage measurements made on the boundary. An EIT system consists of three parts, i.e. array electrode, data acquisition system and image reconstruction unit, as shown in Fig. 1.

2.1. Forward and inverse problems

EIT is composed of forward problem and inverse problem. The forward problem is to determine the voltage measurements, i.e. voltage vector $\boldsymbol{U}$ for a known conductivity distribution $\boldsymbol{\sigma}$ and

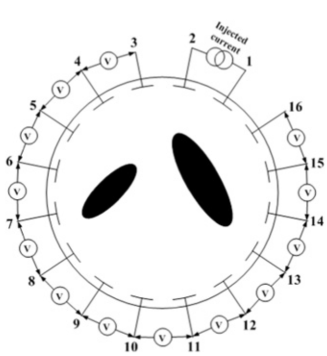

Fig. 1. A sketch-map of EIT sensor.

injected current vector $\boldsymbol{I}$. The inverse problem is also called image reconstruction. The aim of inverse problem for EIT is to obtain the conductivity distribution $\boldsymbol{\sigma}$ using the boundary voltage vector $\boldsymbol{U}$ and injected current vector $\boldsymbol{I}$. In inverse problem, a forward model is used to predict observations. In the specific case of EIT, a model that predicts the spatial electric field resulting from applying a current to a known conductivity distribution is required. The capability to calculate the electric fields within an object also proves an efficient method to assemble the Jacobian matrix which is necessary to solve the inverse problem.

In order to obtain a forward model and the function of $\boldsymbol{U}$ for EIT, the boundary conditions have to be determined. The boundary conditions arise from the current injection and voltage measurements through the boundary electrodes. Commonly these boundary conditions are called electrode models. In this paper, complete electrode model is used [16].

For mathematical model, the complete electrode model is used. The complete electrode model is defined by Laplace's equation

$$\nabla\cdot(\sigma\nabla u)=0 \text{ in } \Omega \tag{1}$$

and the following boundary conditions:

$$u+z_l\sigma\frac{\partial u}{\partial \hat{\boldsymbol{n}}}=U_l, \text{ on } E_l, \quad l=1,2,\ldots,m \tag{2}$$

$$\int_{E_l}\sigma\frac{\partial u}{\partial \hat{\boldsymbol{n}}}d\Gamma=I_l \quad l=1,2,\ldots,m \tag{3}$$

$$\sigma\frac{\partial u}{\partial \hat{\boldsymbol{n}}}=0 \text{ on } d\Gamma\backslash\cup_{l=1}^{m}E_l \tag{4}$$

In these equations σ is the conductivity distribution, u is the scalar potential distribution, $\hat{\boldsymbol{n}}$ is the outward unit normal of the boundary $\partial\Omega$, z_l is the contact impedance, I_l is the injected current and U_l is the corresponding potentials on the electrodes, m is the number of electrodes, E_l is the lth electrode, and Ω denotes the object.

In addition, the following two conditions for the conservation of charge are needed to ensure the existence and uniqueness of the solution

$$\sum_{l=1}^{m}I_l=0 \tag{5}$$

$$\sum_{l=1}^{m}V_l=0 \tag{6}$$

In order to solve the complete electrode model, numerical techniques are preferable to analytic solution because the complexity of obtaining analytic solution usually prevents its application in the forward model. The finite element method (FEM) is widely employed in current EIT forward model. After the FEM discretization, the relation between the injected currents and the measured voltages on the electrodes, i.e. the function of $\boldsymbol{U}$ can be defined based on Eqs. (1)–(4), [17]

$$\boldsymbol{U}=V(\boldsymbol{\sigma};\boldsymbol{I}) \tag{7}$$

where vector $\boldsymbol{\sigma}\in R^{n\times1}$ is the discrete conductivity distribution, and vector $\boldsymbol{U}\in \boldsymbol{R}^{n\times1}$ is the discrete measured voltages. m is the number of measurement data and n is the number of pixels in the reconstructed image. $V(\sigma;I)$ is the forward model mapping the conductivity distribution σ and injected current vector $\boldsymbol{I}$ to the boundary voltage vector $\boldsymbol{U}$.

Difference imaging is used in this paper. The aim of difference imaging is to reconstruct the change in conductivity that occurs over some time interval. A data set $\boldsymbol{U}_1$ is acquired at a time t_1 and a second data set $\boldsymbol{U}_2$ is acquired at a later time t_2. The algorithm then calculates the change conductivity from time t_1 to time t_2.