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Extensions of goal-oriented error estimation methods to simulations of highly-nonlinear response of shock-loaded elastomer-reinforced structures

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Abstract

This paper describes extensions of goal-oriented methods for a posteriori error estimation and control of numerical approximation to a class of highly-nonlinear problems in computational solid mechanics. An updated Lagrangian formulation of the dynamical, large-deformation response of structures composed of strain-rate-sensitive elastomers and elastoplastic materials is developed. To apply the theory of goal-oriented error estimation, a backward-in-time dual formulation of these problems is derived, and residual error estimators for meaningful quantities of interest are established. The target problem class is that of axisymmetric deformations of layered elastomer-reinforced shells-of-revolution subjected to shock loading. Extensive numerical results on solutions of representative problems are given. It is shown that extensions of the theory of goal-oriented error estimation can be developed and applied effectively to a class of highly-nonlinear, multi-physics problems in solid and structural mechanics.

Keywords: Nonlinear continuum mechanics; Shock loading; Goal-oriented error estimation; A posteriori error estimation; Dual problem

1. Introduction

We revisit a subject addressed nearly a half-century ago by John Argyris: ''Continua and Discontinua'', where some of the early finite element approximations of a large class of nonlinear problems in continuum mechanics were presented [2]. Our goal here is to bring to the toolkit a new methodology for problems in nonlinear continuum mechanics: goal-oriented a posteriori error estimation for highly-nonlinear dynamic simulations of the deformation of submerged bodies subjected to shock loading.

Methods for developing a posteriori estimates of finite element approximations of linear elliptic boundary-value problems first appeared in the literature in the late 1970s, beginning with the work of Ladevèze [11] for elasticity problems and of Babuška and Rheinboldt [3] on two-point boundary-value problems and followed in the early 1980s by extensions to elliptic problems on two- and three-dimensional domains [4,5,16]. Until the mid 1990s, except maybe the work of Gartland [8], virtually all of the methods of error estimation, an essential ingredient in mesh adaptation techniques, were applicable to global estimates of error in finite element approximations of linear boundary-value problems. A review of a posteriori error estimation can be found in the monograph of Ainsworth and Oden [1]. Global estimates for certain classes

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of nonlinear elliptic problems were contributed by Verfürth [24], see also [1]. More recently, techniques for developing a posteriori error estimates of so-called quantities-of-interest which are functionals of solutions of linear PDEs, were presented by Oden and Prudhomme [17,18] and Becker and Rannacher [6]. These techniques employ optimal control strategies and involve the solution of a dual problem in which the quantity of interest appears as data. A variety of applications of these ideas have appeared since 2000 (e.g. [20–23]). The paper of Becker and Rannacher [6] appearing in 2001 extended the dual-based theory of error estimation to nonlinear boundary- and initial-value problems, and the paper of Oden and Prudhomme [18] extended the theory further to cover estimation of both modeling and approximation error in 2002. Further extension of this work to multi-scale modeling methods is described in a recent report [19].

While the developments to date provide an abstract mathematical framework for error estimation in highly-nonlinear problems, few applications to important problems in nonlinear continuum mechanics have been made [10,12,13], owing to the inherent complexities in such problems. To capture features of the nonlinear dynamics of solid bodies and structures under shock loading involves a host of complicated features and has been the focus of research in computational solid mechanics for many decades (see [15] or the more recent treatise [7]). The analysis of the evaluation of approximation error of quantities of interest in such applications involves solving first a forward-in-time problem for the system response, and then a backward-in-time problem for the dual solution associated with the particular quantity of interest.

In the present investigation, a posteriori error estimates for key quantities of interest are derived for a class of complex and highly-nonlinear problems in computational solid mechanics: the dynamical behavior of a heterogeneous, layered shells subjected to shock loading. The models considered here involve axisymmetric deformations of thick bodies-of-revolution undergoing very high strains and strain rates, and large elastic and inelastic deformations. The target applications of the methods developed in this investigation is the dynamical behavior of elastomer-reinforced steel shells subjected to high-intensity shock loading.

Following the introduction, the underlying theory of a posteriori error estimation is given in Section 2. The formulation of a continuum model of the problem is given in Section 3. Here a framework suitable for goal-oriented error estimation is presented and algorithms used to compute the solution of the discretized space and time problem are established. Section 4 presents the equations of goal-oriented error estimation and the particular solution technique used. The method of assessing the fidelity of the solution is also discussed. The geometry and data of the target application problems are given in Section 5 and Section 6 presents the constitutive equations used in the computational model. Detailed numerical results are presented and discussed in Section 7. Concluding comments are collected in Section 8.

2. Goal-oriented a posteriori error estimation and control

The theory of goal-oriented error estimation and control can be described in terms of the abstract problem,

$$\boxed{\begin{aligned}&\text{Find } u \in \mathscr{V} \text{ such that}\\ &B(u;v) = F(v) \quad \forall v \in \mathscr{V},\end{aligned}} \tag{1}$$

where $B(\cdot,\cdot)$ is a semi-linear form, nonlinear in the first entry, v is a test vector, $F(\cdot)$ a linear functional, and $\mathscr{V}$ is the space of admissible solutions, here a Banach space with norm $\|\cdot\|_{\mathscr{V}}$. Of interest is the value of a functional $Q:\mathscr{V}\to\mathbb{R}$ at solutions u to (1); the quantity of interest. The problem of determining $Q(u) = \inf\{Q(v) : v \in \mathscr{V}\}$ subject to (1) is an optimal control problem characterized by the pair of equations:

$$\boxed{\begin{aligned}&\text{Find } (u,p) \in \mathscr{V}\times\mathscr{V} \text{ such that}\\ &B(u;v) = F(v) \quad \forall v \in \mathscr{V}\\ &B'(u;w,p) = Q'(u;w) \quad \forall w \in \mathscr{V}.\end{aligned}} \tag{2}$$

Here

$$\begin{aligned}B'(u;w,p) &= \lim_{\theta\to 0}\frac{1}{\theta}[B(u+\theta w;p) - B(u;p)],\\ Q'(u;w) &= \lim_{\theta\to 0}\frac{1}{\theta}[Q(u+\theta w) - Q(u)].\end{aligned} \tag{3}$$

Problem $(2)_1$ is the dual problem associated with the quantity of interest Q and the primal problem (1) (or $(2)_2$). The dual problem is thus linear in p, but coupled to the possibly nonlinear primal problem.

We next consider a family $\{\mathscr{V}^h\}$ of finite-dimensional subspaces of $\mathscr{V}$ with everywhere dense union

$$\bigcup_{h\to 0}\mathscr{V}^h$$