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Synchronization of chaotic systems with known and unknown parameters using a modified active sliding mode control

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ABSTRACT

This paper defines a new surface in an active sliding mode to synchronize two chaotic systems with parametric uncertainty. To verify the capability of the proposed scheme, signals are also contaminated by measurement noise. The integral acting surface produces a dynamics for error, where the appropriate eigenvalues are easily assigned. Using this surface, calculation of parameters of the controller becomes simpler than the classical alternative. A sufficient condition, as a guideline of the designated procedure, is dedicated to provide a robust stability of the error dynamics. Finally, a simulation study is performed to verify the robustness and effectiveness of the proposed control strategy.

1. Introduction

In recent years, the chaos theory and relevant properties have found useful applications in many engineering areas such as secure communication, biological systems, power electronic devices and power quality, digital communication, chemical reaction analysis, … [1–11]. In 1990 Pecora and Caroll [1] introduced a method for synchronizing two identical chaotic systems with different initial conditions. They reported that chaotic systems possess a self-synchronization property.

A typical configuration of chaotic synchronization consists of drive and response systems. A drive system propels the response system via a coupling signal to achieve the synchronization of coupled chaotic systems. A variety of alternative schemes have been proposed in the literature to ensure control and synchronization of such systems. Their potential was used to create secure communication systems [10].

The design procedure of an active sliding mode controller is a combination of an active controller and a sliding mode one. During some active sliding mode design procedures, there are some parameters of the controller which are needed to be determined. However, determination of the parameters is a somewhat exhaustive task. In this paper, a new surface designation will be introduced to cope with this problem.

This paper is organized as follows. In Section 2, an active sliding mode control is introduced. In Section 3, the theory of the active sliding mode and the proof of the stability of the controller with uncertain models and noisy perturbed states are presented. Numerical simulations are given in Section 4 to illustrate the effectiveness of the proposed method. Finally, a conclusion in Section 5 closes the work.

2. System description and the problem formulation

Let us define the following two 3-dimensional uncertain chaotic systems as master and slave, respectively by:

$$\dot{x} = (A_1 + \Delta A_1)x + g_1(x) + \Delta g_1(x) \tag{1}$$

$$\dot{y} = (A_2 + \Delta A_2)y + g_2(y) + \Delta g_2(y) + u(t) \tag{2}$$

where $x(t) \in R^3$ and $y(t) \in R^3$ denote state vectors of the system. A_1 and $A_2 \in R^{3\times 3}$ represent the linear parts of the system dynamic, and $g_1 : R^3 \to R^3$ and $g_2 : R^3 \to R^3$ are the nonlinear parts of the system. $\Delta A_1 \in R^{3\times 3}$ and $\Delta A_2 \in R^{3\times 3}$ are unknown linear parts of matrices. $\Delta g_1 : R^3 \to R^3$ and $\Delta g_2 : R^3 \to R^3$ are unknown nonlinear parts of the master and slave systems respectively. To synchronize the state $y(t)$, with the state of the master system $x(t)$, the controller $u(t) \in R^3$ has been added to the slave system. The synchronization problem is to design the controller $u(t)$ which synchronizes the states of the slave with that of the master. However, the synchronization goal is as follows:

$$\lim_{t\to\infty} \|x(t) - y(t)\| \to 0 \tag{3}$$

where $\|.\|$ is the Euclidean norm (2-norm) of the vector.

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3. Methodology of active sliding mode control design

3.1. Sliding surface

Let the synchronization error be defined as $e = y - x$. Using systems (1) and (2) immediately gives the synchronization error as:

$$\dot{e} = A_2 y + \Delta A_2 y + g_2(y) + \Delta g_2(y) - A_1 x - \Delta A_1 x - g_1(x) - \Delta g_1(x) + u(t). \tag{4}$$

The sliding surface can be chosen as:

$$s(e) = e - (K + A_2)\int_0^t e\mathrm{d}t \tag{5}$$

where K is a constant gain matrix.

3.2. Active sliding mode controller design

According to active control design strategy [10–18], control input $u(t)$ can be used as follows:

$$u(t) = H(t) - g_2(y) + g_1(x) - (A_2 - A_1)x \tag{6}$$

in which $H(t)$ is designed based on a sliding mode control law. Although, there are many possible choices for the control $H(t)$, without loss of generality, the sliding mode control law is chosen by:

$$H(t) = Kw(t). \tag{7}$$

Here $w(t) \in R$ is a control input and can be determined as:

$$w(t) = \begin{cases} w^+(t) & s(e) \geq 0 \\ w^-(t) & s(e) < 0 \end{cases} \tag{8}$$

where $s = s(e)$ is a switching surface which introduces the desired dynamics. Replacing $u(t)$ in Eq. (4), into Eq. (6), the error dynamic is yielded as:

$$\dot{e} = Kw(t) + A_2 e + M(x, y) \tag{9}$$

in which $M(x, y)$ represents the uncertain part of the dynamic which is given by:

$$M(x, y) = \Delta A_2 y + \Delta g_2(y) - \Delta A_1 x + \Delta g_1(y). \tag{10}$$

In this part, let us assume $M = 0$ where in the next section, it will be extended for $M \neq 0$. An equivalent control approach will be found by $\dot{s}(e) = 0$. A necessary condition for the state trajectory to stay on the switching surface is as:

$$s(e) = 0 \tag{11}$$

together with:

$$\dot{s}(e) = 0. \tag{12}$$

Now, using (5) alters Eq. (9) to:

$$\dot{s}(e) = \dot{e} - (K + A_2)e = K(w - e) = 0. \tag{13}$$

Solving (13) for $w(t)$ gives the equivalent control $w_{\mathrm{eq}}(t)$ which is as follows:

$$w_{\mathrm{eq}} = e. \tag{14}$$

Replacing (14) in (9), the error dynamic of the sliding mode will be given as:

$$\dot{e} = (K + A_2)e. \tag{15}$$

K is tuned such that all eigenvalues of $K + A_2$ have negative real parts, hence the system is asymptotically stable. A constant plus proportional rate reaching law is of concern here [10]. Then, the reaching phase law can be chosen such that:

$$\dot{s} = -q\,\mathrm{sgn}(s) - rs \tag{16}$$

in which, $\mathrm{sgn}(s)$ stands for the signum function. Gains $r > 0$ and $q > 0$ are determined such that the sliding condition is met and the sliding mode motion occurred. From (13) and (16) and replacing for s from (5), one can obtain:

$$w(t) = K^{-1}(-q\,\mathrm{sgn}(s) - rs + Ke). \tag{17}$$

3.3. Robust stability analysis

In order to verify the stability of the above controlled system, the following Lyapunov function is candidate:

$$V = \frac{1}{2}s^2 \tag{18}$$

where the time derivative of (18) is given by:

$$\dot{V} = s\dot{s} = -qs\,\mathrm{sgn}(s) - rs^2. \tag{19}$$

Since $s\,\mathrm{sgn}(s) > 0, r > 0$ and $q > 0$ immediately gives $\dot{V} < 0$, therefore $\dot{V}$ is negative definite. Consequently, the switching surface s is bounded and the surface s asymptotically converges to zero. Substituting (17) in (9), we obtain the error dynamics as:

$$\dot{e} = (A_2 + K)e - q\,\mathrm{sgn}(s) - rs. \tag{20}$$

As a linear system with bounded input, the error system is asymptotically stable if and only if $(A_2 + K)$ has negative eigenvalues. Since s is asymptotically stable the error dynamics are also asymptotically stable i.e. $\lim_{t\to\infty} \|e(t)\| \to 0$.

- *The principle*:

With respect to the error dynamic (15), the sliding surface is developed to be defined as follows:

$$s(e) = e + P\int_0^t e\mathrm{d}t$$

where P denotes a matrix gain as $P \in R^{3\times 3}$. The primary goal is to obtain matrix P to satisfy the error dynamics in (15). Differentiating the surface $s(e)$ with respect to the error in (9), yields:

$$Kw_{\mathrm{eq}} + A_2 e + Pe = 0. \tag{21}$$

Given (14) and replacing it into (21), P will become:

$$P = -(K + A_2).$$

Therefore the sliding surface (5) is derived.

3.4. Stability analysis of uncertain chaotic systems

In this part, the stability is analyzed for combination of master and slave systems. This will be performed when two different linear and nonlinear uncertain parts ($M(x, y) \neq 0$) are included. Replacing (17) in (9) makes the error dynamic:

$$\dot{e} = (K + A_2)e - q\,\mathrm{sgn}(s) - rs + M(x, y). \tag{22}$$

If unknown nonlinear parts $\Delta g_1(x)$ and $\Delta g_2(x)$ are Lipschitz,[1] then:

$$|M(x, y)| \leq N|e| + B|x| \tag{23}$$

where:

$$N = (\|\Delta A_2\| + L_2) \quad \text{and} \quad B = (\|\Delta A_2 - \Delta A_1\| + L_2 + L_1). \tag{24}$$

In order to design a robust controller, the Lipschitz[1] condition must be satisfied which needs uncertainties (and of course B and N) are bounded, i.e. $\|B\| \leq \beta I$ and $\|N\| \leq \eta I$. Since states of the master system are bounded, therefore $M(x, y)$ will be linearly bounded [10].

Proof. Let us consider the following Lyapunov function:

$$V = \frac{1}{2}s^2. \tag{25}$$

The time derivative of (25) is:

$$\dot{V} = s\dot{s}. \tag{26}$$

[1] Lipschitz coefficients of L_1 and L_2, means: $\|\Delta f_i(x)\| \leq L_i\|x\|, i = 1, 2$.