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## $H_\infty$ decentralized observation and control of nonlinear interconnected systems

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#### ABSTRACT

In this paper, we investigate the problem of  $H_{\infty}$  decentralized tracking control design with a decentralized observer for interconnected nonlinear systems which are characterized by the interconnection of N subsystems. Each subsystem is modeled by a linear constant part perturbed by an additive nonlinearity which is illustrated by the interconnection terms.

The proposed feedback control scheme is developed to ensure the asymptotic stability of the augmented system, to reconstruct the non-measurable state variables of each subsystem, to maximize the nonlinearity domain, and to improve the performance of the model reference tracking control by using the  $H_{\infty}$  criterion despite the external disturbances.

The proposed control approach is formulated in a minimization problem and derived in terms of linear matrix inequalities (LMIs) whose resolution yields the decentralized control and observation gain matrices.

The effectiveness of the proposed control scheme is demonstrated through numerical simulations on a power system with three interconnected machines.

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#### 1. Introduction

The control of linear and nonlinear interconnected systems has become an area of significant research due to its various applications such as aerospace engineering, chemical processes, power systems, transportation networks and others. These processes are characterized by complex nonlinear dynamic models with varying parameters and can be the field of important perturbations. Although it is not obvious, decentralized control schemes are required to achieve the stabilization of these systems under different operating conditions and different system configurations.

The field of decentralized control of nonlinear interconnected systems, where the information exchange between subsystems is not needed, has seen considerable progress and has been applied in numerous works such as [1–3]. Robust decentralized control has been applied to stabilize nonlinear interconnected systems under model parametric perturbations; see, for instance [4–6]. In addition, decentralized adaptive control and decentralized output feedback control schemes have also been presented [7–10]. Recently, much research has been devoted to fuzzy decentralized

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control in order to guarantee the stability, the robustness and the control performance issues of fuzzy systems. In these approaches, the T-S method is used to model the nonlinear interconnected systems [11–13].

All the decentralized control methods mentioned previously deal with the state feedback control [14-17]. However, state variables are often not fully available at each subsystem and sometimes are difficult or costly to measure. Consequently, it may be possible to use an observer to estimate the unknown states [18-20].

In the literature, several works concern the state observation of interconnected systems. The interconnected observer, where the interconnection terms can be used in the observer design, is proposed in [21–23]. On the other hand, the decentralized observer and decentralized observer based feedback control, which depend only on the subsystem local inputs and outputs, present an other attempt at designing observers for use in the feedback stabilization for interconnected systems [24,25].

However, the synthesis of a decentralized observer is not an obvious problem since the separation principle may not be applicable in this situation [26]. To overcome this problem, we consider simultaneously the problem of control and observer design to ensure the global asymptotic stabilization of the interconnected systems. In [27], a decentralized output feedback controller for a class of interconnected nonlinear systems was considered. In this work, the observer and the control gain matrices, for each individual subsystem, are obtained by the resolution of the Riccati equation. In [28,29], a decentralized observer based output feedback control for large-scale systems

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with quadratic interconnections is developed as an optimization problem using LMIs [30].

The contribution of this paper is the development of a decentralized observer based  $H_{\infty}$  decentralized model reference tracking control approach to ensure the asymptotic stabilization with some prescribed constraints of the interconnected system, to permit the reconstruction of the non-measurable state variables, to maximize the nonlinear domain illustrating the interconnection functions with quadratically bounded nonlinear interconnections, and to guarantee the  $H_{\infty}$  performances allowing the minimization of the external disturbance effects. The proposed control approach is designed by an optimization problem given in LMI terms to calculate the control and observation gain matrices.

This paper is organized as follows. The problem is formulated in Section 2. Then Section 3 is devoted to studying the stability of the augmented system in the first part and to developing the decentralized observer based  $H_{\infty}$  decentralized tracking control approach in terms of LMI optimization problem in the second part. In Section 4, numerical simulations on power systems with three interconnected machines are given to highlight the efficiency of the proposed approach. Finally, some conclusions are provided in Section 5.

#### 2. Problem formulation

This section is devoted to the development of a decentralized observer based  $H_{\infty}$  tracking control approach for large-scale interconnected nonlinear systems. Within this framework, we consider an interconnected system described by the interconnection of *N* subsystems as follows:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + h_i(t, x(t)) + w_i(t) \\ y_i(t) = C_i x_i(t) + v_i(t); \quad i = 1, \dots, N \end{cases}$$
(1)

where

- $x_i(t) \in \mathbb{R}^{n_i}$  is the state vector of the *i*th subsystem;
- $u_i(t) \in \mathbb{R}^{m_i}$  is the control vector of the *i*th subsystem;
- $y_i(t) \in \mathbb{R}^{p_i}$  is the output vector of the *i*th subsystem;
- $w_i(t)$  is the external disturbance of the *i*th subsystem;
- $v_i(t)$  is the measurement noise of the *i*th subsystem;
- *h<sub>i</sub>*(*t*, *x*(*t*)) = *h<sub>i</sub>* designates the interconnection terms vector characterizing the nonlinearity of the subsystem;
- $A_i \in R^{n_i \times n_i}, B_i \in R^{n_i \times m_i}, C_i \in R^{p_i \times n_i}$  are respectively the state matrix, the input matrix, and the output matrix of each subsystem;
- (*A<sub>i</sub>*, *B<sub>i</sub>*) is controllable;
- $(A_i, C_i)$  is observable.

The state representation of the global system is given by the following expression:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + h(t, x(t)) + w(t) \\ y(t) = Cx(t) + v(t) \end{cases}$$
(2)

where

- $x^{T}(t) = x^{T} = [x_{1}^{T}, x_{2}^{T}, \dots, x_{N}^{T}]$  is the state vector of the global system;
- $u^{T}(t) = u^{T} = [u_{1}^{T}, u_{2}^{T}, \dots, u_{N}^{T}]$  is the control vector of the global system;
- $\mathbf{y}^{\mathrm{T}}(t) = \mathbf{y}^{\mathrm{T}} = [\mathbf{y}_{1}^{\mathrm{T}}, \mathbf{y}_{2}^{\mathrm{T}}, \dots, \mathbf{y}_{N}^{\mathrm{T}}]$  is the output vector of the global system;
- $\tilde{w}^{T}(t) = w^{T} = [w_{1}^{T}, w_{2}^{T}, \dots, w_{N}^{T}]$  is the disturbance vector of the global system;
- $v^{\mathrm{T}}(t) = v^{\mathrm{T}} = [v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, \dots, v_N^{\mathrm{T}}]$  is the noise vector of the global system;
- $h^{T}(t, x(t)) = h^{T} = [h_{1}^{T}, h_{2}^{T}, \dots, h_{N}^{T}]$  describes the nonlinear interconnection functions vector;

- $A = \operatorname{diag}(A_i), B = \operatorname{diag}(B_i), C = \operatorname{diag}(C_i);$
- the pair (*A*, *B*) is controllable and the pair (*A*, *C*) is observable. We assume that the nonlinear functions *h<sub>i</sub>*(*t*, *x*(*t*)) are uncer-

tain and that they satisfy the following inequality:

$$\|h_i(t, \mathbf{x}(t))\| \leq \alpha_i \|\mathbf{x}(t)\| \tag{3}$$

where

- $\alpha_i$ , i = 1, ..., N, are bounding parameters to be maximized;
- ||.|| is the Euclidean norm.

The reference model for the *i*th subsystem can be designed as follows:

$$\dot{\mathbf{x}}_{ri}(t) = A_{ri}\mathbf{x}_{ri}(t) + r_i(t) \tag{4}$$

where

- $x_{ri}(t) \in \mathbb{R}^{n_i}$  is the reference state vector of the *i*th subsystem;
- $A_{ri} \in R^{n_i \times n_i}$  is the reference state matrix, which must be asymptotically stable;
- $r_i(t) \in \mathbb{R}^{n_i}$  denotes a bounded reference input.

The state variables  $x_i(t)$  are required to follow the trajectories of the reference state variables  $x_{ri}(t)$  for all  $t \ge 0$ .

The reference model for the global system can be written by the following equation:

$$\dot{x}_r(t) = A_r x_r(t) + r(t) \tag{5}$$

where

- $x_r^{\mathrm{T}}(t) = x_r^{\mathrm{T}} = [x_{r1}^{\mathrm{T}}, x_{r2}^{\mathrm{T}}, \dots, x_{rN}^{\mathrm{T}}]$  is the reference state vector of the global system;
- $r^{T}(t) = r^{T} = [r_{1}^{T}, r_{2}^{T}, \dots, r_{N}^{T}]$  is the bounded reference input of the global system;
- $A_r = \text{diag}(A_{ri})$  is the reference state matrix of the global system.

The observed state vector of each subsystem, which is a function only of the input and the output of the considered subsystem, can be described by the following equations:

$$\begin{aligned}
\hat{x}_{i}(t) &= A_{i}\hat{x}_{i}(t) + B_{i}u_{i}(t) + L_{i}[y_{i}(t) - \hat{y}_{i}(t)] \\
\hat{y}_{i}(t) &= C_{i}\hat{x}_{i}(t); \quad i = 1, \dots, N
\end{aligned}$$
(6)

with  $L_i$  the observation gain matrix of the *i*th subsystem.

The state observer of the global system, composed by the *N* local observers, can be expressed as follows:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L[y - \hat{y}] \\ \dot{y} = C\hat{x} \end{cases}$$
(7)

with  $\hat{x}^{T} = [\hat{x}_{1}^{T}, \hat{x}_{2}^{T}, \dots, \hat{x}_{N}^{T}]; L = \text{diag}(L_{i})$  is the block diagonal observation gain matrix.

Notice that the state observation structure of the global interconnected nonlinear system is completely decentralized since there is no information transfer between local observers.

The observation error between the real state and the observed one,

$$e_i(t) = e_i = x_i - \hat{x}_i,\tag{8}$$

is described by the following differential equation:

$$\dot{e}_i = [A_i - L_i C_i] e_i + h_i + w_i - L_i v_i.$$
(9)

The error observation,  $e(t) = e = x(t) - \hat{x}(t)$ , of the global system, can then be designed as follows:

$$\dot{e} = [A - LC]e + h + w - Lv. \tag{10}$$

The local control law of each subsystem is given by

$$u_i = -K_i[\hat{x}_i(t) - x_{ri}(t)]$$
(11)

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