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Use of uncertainty polytope to describe constraint processes with uncertain time-delay for robust model predictive control applications

Gongsheng Huang *, Shengwei Wang

Department of Building Services Engineering, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

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ABSTRACT

This paper studies the application of robust model predictive control (MPC) in a constraint process suffering from time-delay uncertainty. The process is described using a transfer function and sampled into a discrete model for computer control design. A polytope is firstly developed to describe the uncertain discrete model due to the process's time-delay uncertainty. Based on the proposed description, a linear matrix inequality (LMI) based MPC algorithm is employed and modified to design a robust controller for such a constraint process. In case studies, the effect of time-delay uncertainty on the control performance of a standard MPC algorithm is investigated, and the proposed description and the modified control algorithm are validated in the temperature control of a typical air-handling unit.

1. Introduction

Time-delay is a typical nonlinearity and exists universally in the dynamic behaviors of many real systems. It happens probably due to mechanic problems, internal physical phenomenon or computational delay. Approximation of high-order systems using low-order models may also result in time-delay. Time-delay is an important issue in controller design and a number of time-delay compensation strategies have been developed, such as Smith Predictor and Internal Model Control, to improve the control performance [1]. Model-based predictive control (MPC), a practical computer control technique, has also been used to deal with the control problem of systems suffering from time-delay in that it can cope with constraints simultaneously [2,3].

Most MPC algorithms use a state-space discrete model to describe a continuous system and optimize the control input according to the predicted outputs or states [4]. When a MPC algorithm is used to control time-delay systems, the length of the time-delay is usually assumed to be known and fixed [2,3]. In practical applications, however, it is sometimes difficult to know exactly the length of the time-delay since time-delay may vary with the process operating environment. For example, time-delay variations can be observed in air-conditioning systems due to thermo-fluid processes [5,6]. Time-delay mismatch may have a significant effect on the closed-loop response of MPC because the control performance of MPC depends much on the accuracy of the predictive model. Therefore, it is necessary to allow for time-delay uncertainties directly in the MPC algorithm design when time-delay uncertainties occur and seriously affect the control performance.

Generally, the dynamics of a constraint process can be described by

$$\frac{y(s)}{u(s)} = \frac{\beta_1 s^{n-1} + \cdots + \beta_{n-1} s + \beta_n}{s^n + \alpha_1 s^{n-1} + \alpha_{n-1} s + \alpha_n} e^{-\tau s} = G(s)\, e^{-\tau s} \tag{1}$$

$$\tau \in \left[\underline{\tau}, \bar{\tau}\right] \tag{2}$$

$$|\dot{u}| \le \delta s^{-1} \tag{3}$$

$$\underline{D} \le u \le \bar{D} \tag{4}$$

where $G(s)$ is a transfer function and τ is the time-delay. The length of the time-delay is unknown or time-varying, but it lies in the uncertainty set defined by Eq. (2). Eq. (3) denotes a rate limit on the control input and Eq. (4) defines the operating range of the control input. When the continuous model described by Eq. (1) is sampled into a discrete model for computer control design, the discrete model is also uncertain. In this paper, an uncertainty polytope will be developed to describe the uncertain discrete model.

* Corresponding address: Department of Building Services Engineering, Hong Kong Polytechnic University, Room 1202, New East Ocean Centre 9 Science Museum Road, Hung Hom, Tsim Sha Tsui District, Kowloon, Hong Kong. Tel.: +852 34003601; fax: +852 27746146.

E-mail addresses: huang.gongsheng@polyu.edu.hk (G. Huang), beswwang@polyu.edu.hk (S. Wang).

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Uncertainty polytope has been widely used in the control literature to describe uncertain models [2,4,7]. This type of uncertainty description defines a polytope in which the parameters defining the model must lie. It assumes that the parameters at the corners of the polytope are known and the real system lies in which is the convex hull of these corners [4]. For a discrete state-space model

$$x_{k+1} = \mathrm{A}x_k + \mathrm{B}\Delta u_k \tag{5}$$

the uncertainty polytope for the coefficient matrices A, B is defined as

$$\Omega := \left\{ (\mathrm{A}, \mathrm{B}) = \sum_{i=1}^{L} \lambda_i (\mathrm{A}_i, \mathrm{B}_i), \lambda_i \geq 0 \text{ and } \sum_{i=1}^{L} \lambda_i = 1 \right\} \tag{6}$$

where Ω is the uncertainty polytope; $i = 1, \ldots, L$ denotes the corners of Ω. The models with $(\mathrm{A}_i, \mathrm{B}_i)$ at the corners are known and the real plant may vary with time as long as it remains within this polytope [4].

The integration of the description of time-delay uncertainties into the framework of uncertainty polytope makes it possible to use existing robust MPC strategies to design a robust controller for processes suffering from time-delay uncertainties. Robust MPC is developed from MPC and gained popularity in recent years because it can deal with constraints and uncertainties simultaneously [8–10]. Many methods have been developed to formulate robust MPC that depends on the uncertainty description associated with the predictive model [11,12]. Because time-delay is one of the main causes of performance degradation or instability, robust MPC was widely investigated to deal with the control problem of uncertain time-delay systems [13,14]. However, only a few robust MPC algorithms have been proposed to deal with time-delay uncertainty explicitly. This may be because time-delay uncertainty is unstructured [15], which make it complicated to do prediction based on such uncertain models. Robust predictive control of uncertain systems with time-varying state-delay was studied in [14], which dealt with time-varying state-delay in the framework of linear matrix inequality (LMI) optimization. Robust MPC is used in this paper to design a robust controller for the constraint process described by Eq. (1), which mainly suffers from input/output time-delay uncertainties. Different from many papers on robust MPC which assume that an uncertainty polytope is already known, this paper will show how to develop an uncertainty polytope to describe the time-delay uncertainty in Eqs. (1) and (2). Based on the uncertainty polytope description, an LMI-based MPC algorithm proposed in [2] is employed for control design. Since the original algorithm cannot take account of the constraint (4) directly, a modified scheme is developed, which can improve the robust stability of the controlled process when it suffers from the constraints (3) and (4).

The basic steps of using an uncertainty polytope to describe time-delay uncertainty are illustrated using a first-order plus time-delay model for simplicity. It will then be extended to a more general model in the form of Eq. (1). Case studies are mainly performed on the temperature control of an air-handling unit because it is a typical constraint process suffering from time-delay uncertainties [5]. The paper is organized as follows: Section 2 illustrates how to use an uncertainty polytope to describe the uncertain discrete model sampled from the continuous model (1) with the time-delay uncertainty (2). Section 3 introduces two kinds of extensions: firstly, system-gain uncertainties are integrated into this description; and secondly it is shown that series-connected processes with time-delay and system-gain uncertainties can also be described using the proposed method. Section 4 describes the modification of the LMI-based MPC algorithm for dealing with the constraints (3) and (4). Section 5 presents two case studies: one studies the effect of time-delay uncertainties on the control performance of a standard MPC algorithm; and the other validates the proposed method on the temperature control of a typical air-handling unit. Conclusions are given in Section 6.

2. Using uncertainty polytope to describe time-delay uncertainty

Consider a first-order plus time-delay model

$$\frac{y(s)}{u(s)} = \frac{K}{1+Ts}\mathrm{e}^{-\tau s} \tag{7}$$

where K is the process-gain and T is the time constant. Using a sampling interval h, the continuous model can be discretized as [16]

$$y_{k+1} = (1+a)\, y_k - a y_{k-1} + b_d \Delta u_{k-d} + b_{d+1} \Delta u_{k-d-1} \tag{8}$$

where a, b_d, b_{d+1} are computed by $a = \mathrm{e}^{-h/T}, b_d = K\left(1 - \mathrm{e}^{-(h-\tilde{\tau})/T}\right), b_{d+1} = K\left(\mathrm{e}^{-(h-\tilde{\tau})/T} - \mathrm{e}^{-h/T}\right)$. The discrete time-delay d and $\tilde{\tau}$ satisfy $\tau = dh + \tilde{\tau}, 0 < \tilde{\tau} \leq h$. Note that the sum of b_d and b_{d+1} is not relative to the time-delay τ. This is because

$$b_d + b_{d+1} = K(1-a). \tag{9}$$

When the process is required to track a predefined set-point, the tracking error is denoted as $e_k = y_k - y_r$, where y_r is the output set-point. Eq. (8) can be reformulated as

$$e_{k+1} = (1+a)\, e_k - a e_{k-1} + b_d \Delta u_{k-d} + b_{d+1} \Delta u_{k-d-1}. \tag{10}$$

Define a state vector as $x_k^t = (e_k, e_{k-1}, \Delta u_{k-d-1}, \ldots, \Delta u_{k-1})$, Eq. (10) is rewritten in the form of a state-space model as [16]

$$\begin{aligned} x_{k+1} &= \mathrm{A}_s x_k + \mathrm{B}_s \Delta u_k \\ e_k &= \mathrm{C}_s x_k \end{aligned} \tag{11}$$

where the coefficient matrices $\mathrm{A}_s, \mathrm{B}_s$ and C_s are defined in Appendix A. With an appropriate choice of h, Eq. (10) can be used to substitute the continuous model described by Eq. (7) in computer controller design [12]. It should be noted that the sampling interval h will affect the dimension of the state vector as well as the coefficient matrices. Following the suggestion in [17], the sampling interval is chosen as

$$h \leq T/10. \tag{12}$$

2.1. First-order systems suffering from time-delay uncertainty

When τ varies in the range (2), the uncertainty set for describing the corresponding discrete state-space model sampling from the continuous model (7) is

$$\Omega_{\mathrm{AB}} = \left\{ (\mathrm{A}, \mathrm{B}) = \sum_{i=\underline{d}}^{\bar{d}} \lambda_i (\mathrm{A}_i, \mathrm{B}_i) \right\} \tag{13}$$

where $\mathrm{A}_i, \mathrm{B}_i$ are defined in Appendix B. The dimensions of A_i and B_i are $(\bar{d}+2, \bar{d}+2)$ and $(\bar{d}+2, 1)$ respectively. The coefficients λ_i, $i = \underline{d}, \ldots, \bar{d}$, satisfy

$$\sum_{i=\underline{d}}^{\bar{d}} \lambda_i = 1, \quad 0 \leq \lambda_i \leq 1 \tag{14}$$

$$\lambda_i + \lambda_{i+1} = 1, \quad \forall i \in \left[\underline{d}, \ldots, \bar{d}-1\right]. \tag{15}$$

The uncertainty set Ω_{AB} defined by Eq. (13) is developed as follows. Firstly, any continuous model in the form of Eq. (7) with τ being inside the uncertainty range (2) can be sampled using the sampling interval h into

$$\begin{aligned} M_c : e_{k+1} = {} & (1+a)\, e_k - a e_{k-1} + b_{\underline{d}} \Delta u_{k-\underline{d}} \\ & + b_{\underline{d}+1} \Delta u_{k-\underline{d}-1} + \cdots + b_{\bar{d}} \Delta u_{k-\bar{d}} \end{aligned} \tag{16}$$

where $\underline{d}$ and $\bar{d}$ are defined as $\underline{d} = fl\left(\underline{\tau}/h\right)$ and $\bar{d} = cl\left(\bar{\tau}/h\right)$; and