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Energy-dissipative momentum-conserving time-stepping algorithms for finite strain multiplicative plasticity

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This paper is dedicated to the memory of Professor John Argyris

Abstract

This paper presents a new energy-dissipative momentum-conserving algorithm for multiplicative models of finite strain plasticity. The proposed scheme preserves exactly the conservation laws of linear and angular momenta, and leads to the exact energy dissipation between the two computed solutions in a given time step. In particular, the energy is exactly conserved if the step is elastic, thus recovering existing energy–momentum-conserving schemes in the purely elastic range. Extensions accommodating a strictly non-negative numerical energy dissipation in the high-frequency to handle the numerical stiffness of the problem are also discussed briefly. These conservation/dissipation properties are proven rigorously in the very nonlinear setting of multiplicative finite strain plasticity. In fact, the analysis drives the design of the new return mapping algorithm proposed to integrate the plastic evolution equations so the above dissipation/conservation properties hold. These analyses account for the spatial discretization of the problem in the context of the finite element method. Several representative numerical simulations are presented to illustrate the performance of the new algorithm.

Keywords: Finite strain plasticity; Multiplicative plasticity; Nonlinear dynamics; Time-stepping algorithms; Energy–momentum schemes; Finite elements

1. Introduction

The solution of dynamic problems in solid and structural mechanics has received a great deal of attention over the years. We refer to [9, Chapter 9], for a complete account in the linear range, including, for example, the now classical Newmark family of time-stepping algorithms, dating back to [18]. See also the monograph [1] for a complete account in the context of structural dynamics. However, it is well known by now that these schemes, all developed in the context of linear problems, exhibit serious limitations when applied to geometrically nonlinear problems. The unconditional stability of the schemes is lost, with the numerical simulations showing an unbounded growth of the physical energy during the numerical simulation. We refer to [21,4], among others, for an illustration of these difficulties in the context of nonlinear elastic problems. The same observations apply to the so-called high-frequency dissipative schemes, like the HHT-method of [6] (see [9]), with their dissipative character being lost again in the finite deformation range; see [2,3] for an analysis of these methods in the context again of finite elastic problems.

These limitations extend to general finite deformation elastoplastic problems. The formulation of the aforementioned classical time-stepping algorithms for multiplicative finite strain plasticity can be found in [19]; see also the review article

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[20] and references therein. The lack of the noted conservative/dissipative properties in elastoplastic problems is clear since the elastic problem appears as a particular case. But more concerning than this obvious observation is the fact that the dissipative character of the underlying physical inelastic system does not help in adding numerical stability to the schemes. In fact, we have observed quite the opposite due to the lack of dissipativity of the algorithms employed in the integration of the plastic evolution equations.

To illustrate these ideas, we consider the simple nonlinear system depicted in Fig. 1.1. It consists of an elastoplastic spring fixed at one end and with a mass at the opposite free end. No external forces are applied with the mass evolving in time from some initial position and velocity. Section II.1 in Appendix II presents a complete description of this system, including the full set of governing equations and their conservation/dissipation properties. In particular, the associated angular momentum is conserved (as sketched in Fig. 1.1 by the vector $\boldsymbol{J}$) and the total energy does not increase in time. The total energy of the system consists of the kinetic energy of the mass and the internal energy of the spring, the latter including the elastic strain energy and the hardening plastic potential. In fact, the total energy decreases when the plastic response of the spring is activated, with a full energy conservation during elastic steps. We note the strong nonlinearity of the problem due not only to the finite deformation inelastic response of the spring but also to the large motion (rotation) of the mass around the fixed point.

Fig. 1.2 depicts the solution computed with the standard Newmark scheme known as the trapezoidal rule (or average acceleration method) in combination with a standard return mapping algorithm for the solution of the plastic evolution equations; see Section II.2 in Appendix II for complete details of the discrete equations. The figure shows the trajectory of the mass and the evolution in time of the total energy, the magnitude of the angular momentum and the equivalent plastic strain, a measure of the plasticity occurring in the spring, for different time steps. We clearly observe that the angular momentum is not conserved as in the underlying physical system. The energy response of the algorithm is even more concerning.

The trapezoidal rule is a second-order accurate unconditional (A-)stable scheme in the linear range; see [9, p. 493] . Several studies exist in the literature that have identified the loss of this property in the nonlinear range, with the computed solutions exhibiting a spurious growth of the energy; see e.g. [7,8] and [21]. Although a good asymptotic performance in the energy has also been argued in [7], asymptotic meaning in the limit of very small and very large time steps (i.e. not in practical terms), examples involving a persistent growth of the energy for finite time steps can also be found in this reference. Similarly, pathological examples exhibiting the locking of the computed solution at a non-physical energy level have also been reported in [4]. In a related way, we also refer to the analysis presented in [11] in the context of the so-called variational integrators for elastic problems.

The results presented in Fig. 1.2 clearly confirm that the unconditionally stability of the scheme is lost in the nonlinear inelastic problem considered here. We can observe a sustained growth of the energy for large time steps. Even if the time step size is small, we observe that the evolution of the energy is not monotonic, with increases in the energy of the system. An important observation is the fact that the persistent growth of the energy appears in combination with plastic flow, as the plot of the equivalent plastic strain clearly shows, despite the dissipative character that should be expected just from physical grounds. Furthermore, while the simulations with a large time step show plastic flow in the long-term oscillations of the spring, the same simulations with smaller time steps indicate an elastic response in that range. This clearly shows a

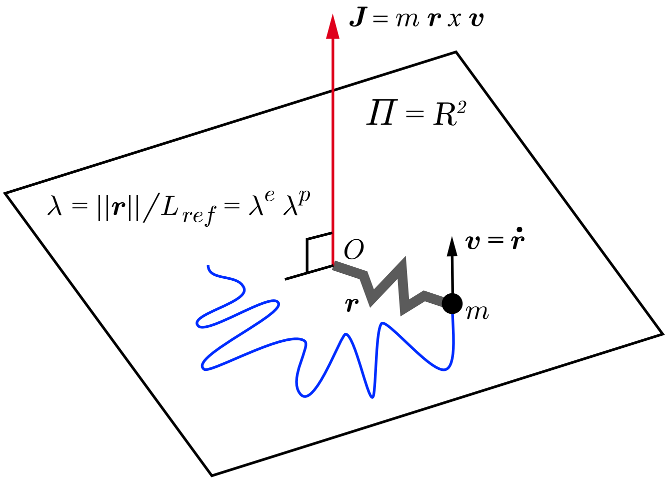

Fig. 1.1. Mass, plastic spring system: problem definition. A finite stretch elastoplastic spring is fixed at one end and with a mass m at the opposite free end. The motion of the mass after some initial conditions is in the plane perpendicular to the conserved angular momentum $\boldsymbol{J}$.