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# Design of nonlinear PID controller and nonlinear model predictive controller for a continuous stirred tank reactor

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ABSTRACT

In this paper, the authors have represented the nonlinear system as a family of local linear state space models, local PID controllers have been designed on the basis of linear models, and the weighted sum of the output from the local PID controllers (Nonlinear PID controller) has been used to control the nonlinear process. Further, Nonlinear Model Predictive Controller using the family of local linear state space models (F-NMPC) has been developed. The effectiveness of the proposed control schemes has been demonstrated on a CSTR process, which exhibits dynamic nonlinearity.



## 1. Introduction

PID controller and linear model predictive controller are the two most popular control schemes that have been widely implemented throughout the chemical process industries for the past two decades. However, control of nonlinear system using above linear control schemes don't give satisfactory performance at all operating points, the reason being that the process parameters of the nonlinear process will vary with the operating conditions. Moreover, the PID controller tuned at one operating condition may not provide satisfactory servo and regulatory performances at shifted operating points. It should be noted that, to achieve improved closed loop performance a different set of controller settings for each operating condition have to be used.

In the case of model based control schemes, the accuracy of the model will have a significant effect on the closed loop performance of the control system. The multiple-linear models concept has been used in the recent years for modeling of nonlinear systems [1]. In addition, multiple-linear model based approaches for controller design [2–5] have attracted the process control community. A plethora of multiple-model adaptive control schemes have been proposed in the control literature [6–9]. Gao et al. [10] has proposed a nonlinear PID controller for CSTR using local model networks. Omar Galan et al. [11] have reported the real-time implementation of multi-linear model based control strategies on the laboratory scale process.

A simple way to describe a nonlinear dynamic system using multiple linear models has been proposed by Takagi–Sugeno [12] and it is being used in this paper to develop Nonlinear PID controller and Nonlinear Model Predictive Controller. The proposed control scheme consists of a family of controllers (Local Controllers) and a scheduler. As suggested by Kuipers and Astrom [13], either local PID controller outputs or the local PID controller parameters can be interpolated. In the case of interpolation of controller parameters, the controllers' structure have to be assumed as homogeneous, whereas interpolation of controllers output does not impose any such constraints. At each sampling instant, the scheduler will assign weights for each controller and the weighted sum of the outputs will be applied as an input to the plant in the case of interpolation of local controller outputs.

As suggested, one can also apply operating regime approaches to develop an operating regime based model that can be applied in a model-based controller [14,15]. Since global information can be applied to determine the control input at each sampling instant, the nonlinear model based controller is expected to achieve better control performance. Recently, stability analysis of a multi-model predictive control algorithm with an application to the control of chemical reactors has been reported by Leyla, Özkan and Kothare, [16].

The key unit operation in chemical plants namely the continuous stirred tank reactor (CSTR) exhibits highly nonlinear dynamic behavior. Hence, there arises a need to develop computationally non-intensive control schemes in order to achieve tighter control of strong nonlinear processes. A plethora of advanced control schemes such as neural adaptive controller [17], nonlinear internal model control scheme [18] and fuzzy model predictive

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**Nomenclature**

| | |
|---|---|
| $C_A$ | Concentration (mol/l) |
| $T$ | Temperature (K) |
| $q_c$ | Coolant flow rate (l/min) |
| $q$ | Feed flow rate (l/min) |
| $C_{A0}$ | Feed concentration (mol/l) |
| $T_0$ | Feed temperature (K) |
| $T_{c0}$ | Inlet coolant temperature (K) |
| $V$ | CSTR volume (l) |
| $hA$ | Heat transfer term (cal/(min K)) |
| $k_0$ | Reaction rate constant (min$^{-1}$) |
| $E/R$ | Activation energy term (K) |
| $-\Delta H$ | Heat of reaction (cal/mol) |
| $\rho$, $\rho_c$ | Liquid density (g/l) |
| $C_p$, $C_{pc}$ | Specific heats (cal/(g K)) |
| $x(k)$ | True state variable |
| $y(k)$ | Measured variables |
| $u(k)$ | Process inputs |
| $A$ | State transition matrix (continuous domain) |
| $B$ | Input matrix (continuous domain) |
| $C$ | Measurement matrix |
| $K_i$ | Steady State gain of the $i$th process model |
| $K_{c,i}$ | Proportional gain of $i$th PID controller |
| $T_{r,i}$ | Integral time of $i$th PID controller |
| $T_{d,i}$ | Derivative time of $i$th PID controller |
| $N_P$ | Prediction horizon |
| $N_c$ | Control horizon |
| $W_E$ | Error weighting matrix (N-MPC) |
| $W_U$ | Controller weighting matrix (N-MPC) |

*Greek letter words*

| | |
|---|---|
| $\Phi$ | State transition matrix (Discrete domain) |
| $\Gamma$ | Input coupling matrix (Discrete domain) |
| $\xi$ | Damping factor |
| $\omega_n$ | Un-damped natural frequency |
| $\lambda$ | Tuning parameter (IMC-PID controller) |

control scheme [19] have been already attempted on the CSTR process which is considered for the simulation study in this paper. Even with the introduction of powerful nonlinear control strategies such as nonlinear internal model control schemes and neural adaptive control, the proposed control schemes remain an attractive control strategy, because it offer advantages such as simple design and low computational complexity.

The main contributions of the paper are as follows: firstly, the nonlinear system is represented as a family of local linear state space models. Secondly, local PID controllers have been designed on the basis of local linear models, the weighted sum of the output from local PID controllers has been used to control the nonlinear process, and finally a nonlinear model predictive control scheme using the family of local linear state space models has been proposed to control nonlinear process.

The organization of the paper is as follows. Section 2 discusses the T–S fuzzy model. Section 3 presents the design of nonlinear PID controller. Section 4 deals with nonlinear model predictive control schemes formulation using local linear models. Section 5 deals with analytical (first principle) model based predictive control formulation. The process considered for simulation study has been discussed in Section 6. Simulation results are presented in Section 7 and the conclusions drawn from the simulation studies in Section 8.

## 2. Takagi–Sugeno (T–S) fuzzy model

Consider a nonlinear system represented by the following nonlinear differential equations:

$$\dot{x} = \bar{f}(x, u, d) \tag{1}$$

$$y = \bar{g}(x, u, d). \tag{2}$$

Eq. (1) describes a deterministic system evolution and can be obtained from the material and energy balances of the process under consideration. Eq. (2) describes the relationships between the measurements and the state variables. In order to describe a discrete nonlinear system, Eqs. (1) and (2) can also be functionally represented in discrete form as

$$x(k) = f\,[x(k-1), u(k-1), d(k-1)] \tag{3}$$

$$y(k) = g\,[x(k-1), u(k-1)] \tag{4}$$

where, $x(k)$ is the system state vector ($x(k) \in R^n$), $u(k)$ is the system input/known deterministic input ($u(k) \in R^m$), $d(k)$ the unmeasured disturbance/unknown input ($d(k) \in R^q$), and $y(k)$ is the measured variable ($y(k) \in R^r$). The parameters $k$ represents the sampling instant and the symbol $f$ and $g$ represent an $n$-dimensional function vectors. We assume that measurements are made at discrete sampling instants with sampling period $T$. Note that the $d(t)$ term described in Eq. (1) is assumed to be piecewise constant for $kT \le t < (k+1)T$

A T–S fuzzy model has been proposed to represent a nonlinear system using locally linearized models [12]. Two different methods for developing a T–S fuzzy model have been suggested in the literature, namely (i) the black box identification via fuzzy clustering technique [20] and (ii) Linearization of an existing nonlinear system around the centers of the fuzzy region partitioning the state space. The T–S fuzzy model is nothing but a piecewise interpolation of local linear models through membership function. The T–S fuzzy model is described by IF-THEN rules, which represent local linear relations of the nonlinear system. The rule to describe the nonlinear system around an operating point is as follows:

Rule $i$ ($i = 1 : N$)

If $z_1(k)$ is $M_{i,1}$ and ... and $z_g(k)$ is $M_{i,g}$ then

$$x_i(k) = \Phi_i(x(k-1) - \bar{x}_i) + \Gamma_i(u(k-1) - \bar{u}_i) \tag{5}$$

$$y_i(k) = C_i x_i(k) \tag{6}$$

where, $z_j(k)$ are the premise variables and $M_{ij}(k)$ are the fuzzy sets. $\Phi_i$, $\Gamma_i$, and $C_i$ are known time invariant matrices of appropriate dimensions. In this work it is assumed that such a model of the process can be developed from the first principles by linearizing them around different operating steady state values ($\bar{x}_i$ and $\bar{u}_i$). The global system behavior is described by a fuzzy fusion of all linear model outputs. For a given input vector, $u(k)$, the global state and output of fuzzy model are inferred as follows:

$$\begin{aligned} x(k) \;=\; & \sum_{i=1}^{N} h_i(z(k))[\Phi_i(x(k-1) - \bar{x}_i) \\ & + \Gamma_i(u(k-1) - \bar{u}_i) + \bar{x}_i] \end{aligned} \tag{7}$$

$$y(k) = Cx(k) \tag{8}$$

where the membership grades $h_i(z(k))$ are defined as

$$h_i(z(k)) = \frac{\mu_i(z(k))}{\mu(k)} \tag{9}$$

$$\mu_i(z(k)) = \prod_{j=1}^{g} M_{ij} \tag{10}$$

$$\mu(k) = \sum_{i=1}^{N} \mu_i(z(k)). \tag{11}$$