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A continuous analysis of multi-input, multi-output predictive control

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Abstract

A continuous formulation and method of analysis is constructed for multi-input, multi-output (MIMO) predictive control and used to compare Dynamic Matrix Control (DMC) with Simplified Predictive Control (SPC). Approximate characteristic equations are derived for each of DMC and SPC and these are used to determine, and thus compare, the closed-loop control behaviour of these methods at times long compared with the sampling time. The MIMO control problem considered is the general case of control over two coupled zones of a first order, linear process where a single control move is simultaneously input into each zone and a single output or measurement, is made from within each zone. The analytical results are illustrated through MIMO control of the terminal composition of a binary distillation column. A practically important result is an analytic basis to understand previous experimental observations that, for a wide range of processes, SPC appears to be as capable as the more sophisticated DMC. Furthermore, it is also shown here that SPC is *well-conditioned* over its entire parameter range in contrast to DMC. This well-conditioned behaviour makes it especially suitable for remote applications where unknown, and variable timing of future moves may be a significant issue.

Keywords: Continuous analysis; Dynamic matrix control; Simplified predictive control; Multi-input multi-output

1. Introduction

Model predictive control, or MPC, is becoming increasingly popular in the research control community and industry. A recent and extensive review may be found in [1–9]. The main reason for the popularity of MPC is its ease of coping with dead time, inverse response, interactions between loops, and constraints in the Multi-Input/Multi-Output (MIMO) implementation.

Most predictive control methods utilize least squares and/or other control move constraints that require the optimization of a cost function over a so-called 'control horizon' that equals the number of control moves computed at each control step (only one move is actually used at each control step). As such, MPC follows a matrix formalism and in its most basic form, is termed 'Dynamic Matrix Control', or 'DMC'. The earliest formal presentation of DMC was in [2] where the basic least squares framework was first presented and upon which most other DMC algorithms have continued to build. However, due to computational complexity and tuning difficulties caused by ill-conditioning, more simplified approaches have also been suggested.

A large simplification, proposed in [5], entirely removed the least squares constraint and reduced DMC to a bare-bones algorithm based upon calculation of the future error at a single location. This 'simplified predictive control' (SPC) has no matrices to consider and is an extreme computational simplication.

The SPC method was originally demonstrated [5] on three experimental examples showing overdamped open-loop responses: (i) a three-input, three-output MIMO control problem without significant delays but with significant interaction between the three zones, (ii) a single-input, single-output (SISO) servo control problem giving an inverse response, and (iii) a two-input, two-output with both measurement and state delays. In all three examples, robustness against severe modelling errors and disturbances was considered and SPC compared well with DMC control under the same conditions. Thus, it was concluded [5] that the ability to handle modelling errors, disturbances, interactions between zones and time delays is uncompromised by SPC.

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However, as stated in [5], reliance in the SPC algorithm upon a future error found at a single location may require more locations for open-loop responses that are underdamped.

A recent industrial study for constrained, online control of multivariable, nonlinear processes and a literature review of SPC, funded by Imperial Oil's University Research Grant programme, is in [6]. The focus of that work was to show that SPC was suited to this class of control application and to give details in the design of the control algorithm and methods to choose control parameters. The method was implemented using a direct approach, usually reserved for matrix predictive methods, where the constraints were embedded in the optimization problem allowing for a direct solution of the optimization problem. The discussion of SPC in [6] was limited to its relative design and computational simplicity in comparison with patented methods such as Quadratic Dynamic Matrix Control (QDMC). Any discussion of results comparing QDMC and SPC on industrial equipment would have required special permission from the QDMC patent holder. Published comparisons of QDMC performance with SPC on smaller, pilot-scale equipment, may be found for example in [7].

These results show that the simplifications made in SPC in comparison with matrix-based predictive control methods do not appear to compromise its performance for a wide range of control applications.

Further analytical work [4] on continuous DMC [2] formulations have gone on to show that, for typical Single-Input/Single-Output (SISO) implementations based upon a control horizon of two moves, SPC actually generalizes the DMC form. In addition, the discrete form of SPC is well-conditioned to parameter changes while DMC is badly conditioned to changes, for example, in sampling time. Practically, this means that although the same discrete control can be obtained from both methods, the well-conditioned behaviour of SPC extends to all control rates. The upshot is that SPC, unlike DMC, is easily tuned through all closed-loop control rates [4]. This unexpected feature, found through the analytical study, means that SPC is suited to control in networked and/or remote control situations where the timing of moves is unknown *a priori* due to unknown communication delays.

The previous discussion brings up the central question of this work: 'How does DMC [2] compare with SPC [5] in the MIMO control situation?' This point *has been thoroughly experimentally investigated* in [5,8] where it was found, for a variety of control situations, that there appears to be little difference in the control results for SPC and DMC control methods. An analytical basis for understanding these experimental results has yet to be found and is the focus of this work. The method used here to compare DMC and SPC, MIMO control is based upon the construction of closed-form, large-time (times much larger than the sampling times) approximations to the continuous DMC and SPC responses via their characteristic equations. The results indicate a larger feature, that matrix-free control formulations that relax the commonly used least-squares constraint, may provide a generic means to circumvent predictive control ill-conditioning *without* sacrificing controllability.

A reduction in ill-conditioning is practically important, as stated earlier, in networked or remote control systems where an ill-conditioned response to changes in sampling times caused by communication delays may unduly compromise control performance.

2. MIMO notation

The multi-input, multi-output closed-loop control application is assumed without loss of generality to involve two zones. Control inputs are found for each zone that account for interactions between them. The linear and time-invariant plant response in the 'Y' Zone is $y(t)$ and in the 'Z' Zone is $z(t)$ and the response to respective accumulated inputs $u(t)$ and $v(t)$ is

$$\begin{aligned} y(t) &= u'(t) * p(t) + v'(t) * q(t) \\ z(t) &= u'(t) * r(t) + v'(t) * s(t) \end{aligned} \tag{1}$$

where '$*$' denotes convolution and the functions $p(t), q(t), r(t)$ and $s(t)$ are open-loop responses. Specifically, $p(t)$ and $r(t)$ are the respective responses in Zone 'Y' and 'Z' to a unit input, $u(t) = 1$, into Zone 'Y' and a zero input, $v(t) = 0$, into Zone 'Z'. On the other hand, $q(t)$ and $s(t)$ are the responses in Zone 'Y' and 'Z' to a zero input, $u(t) = 0$, into Zone 'Y' and a unit input, $v(t) = 1$, into Zone 'Z'.

The closed loop control problem involves the application of piecewise constant control changes, or *moves*, that are held constant for a time Δt between moves. Each new move, Δu_n is applied at time $t_n = n\Delta t$ for a duration Δt. Because the moves are piecewise constant, the process response is naturally restated in a discrete, or 'sampled', form. The sampled plant response in the 'Y' and 'Z' zone is respectively labelled $y^n(t_j) \equiv y^n_j$ and $z^n(t_j) = z^n_j$ where: (i) the superscript implies that the moves $\Delta u_0, \ldots, \Delta u_{n-1}$ and $\Delta v_0, \ldots, \Delta v_{n-1}$ are *past* moves, and (ii) the subscript notation, $j \geq 1$, refers to the *predicted plant response*. Thus, the *current plant response* at time t_{n+0}, the zero in the subscript highlights the choice of $j = 0$, is y^n_0 and z^n_0. Similarly, the *predicted plant response* to these past moves, at times $t_{n+j} = (n + j)\Delta t$, is y^n_j and z^n_j, $j \geq 1$. The discrete form of the plant response in each zone may be written as

$$\begin{aligned} y^n_i &= \sum_{j=1}^{n} \Delta u_{n-j} p_{i+j} + \sum_{j=1}^{n} \Delta v_{n-j} q_{i+j} \\ z^n_i &= \sum_{j=1}^{n} \Delta u_{n-j} r_{i+j} + \sum_{j=1}^{n} \Delta v_{n-j} s_{i+j} \end{aligned} \tag{2}$$

where p_n, q_n, r_n, and s_n are the open-loop tests, $p(t), q(t), r(t)$, and $s(t)$, sampled at times $t = t_n$.

Now, the *predicted errors* of the plant response in each zone, due to *past moves*, are $\hat{\mathbf{e}}^{\mathbf{T}} = (d^n_1, d^n_2, \ldots, e^n_1, e^n_2, \ldots)$. The error components, d^n_i and e^n_i, are the differences in the 'Y' and the 'Z' zone between the setpoint trajectories, $y^n_{sp_i}$ and $z^n_{sp_i}$, and the corresponding plant responses y^n_i and z^n_i. In the standard DMC model, the future move vector is chosen to zero the predicted