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Critical state plasticity. Part VI: Meso-scale finite element simulation of strain localization in discrete granular materials

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Abstract

Development of more accurate mathematical models of discrete granular material behavior requires a fundamental understanding of deformation and strain localization phenomena. This paper utilizes a meso-scale finite element modeling approach to obtain an accurate and thorough capture of deformation and strain localization processes in discrete granular materials such as sands. We employ critical state theory and implement an elastoplastic constitutive model for granular materials, a variant of a model called "Nor-Sand", allowing for non-associative plastic flow and formulating it in the finite deformation regime. Unlike the previous versions of critical state plasticity models presented in a series of "Cam-Clay" papers, the present model contains an additional state parameter ψ that allows for a deviation or detachment of the yield surface from the critical state line. Depending on the sign of this state parameter, the model can reproduce plastic compaction as well as plastic dilation in either loose or dense granular materials. Through numerical examples we demonstrate how a structured spatial density variation affects the predicted strain localization patterns in dense sand specimens.

Keywords: Granular materials; Strain localization

1. Introduction

Development of accurate mathematical models of discrete granular material behavior requires a fundamental understanding of the localization phenomena, such as the formation of shear bands in dense sands. For this reason, much experimental work has been conducted to gain a better understanding of the localization process in these materials [1–11]. The subject also has spurred considerable interest in the theoretical and computational modeling fields [12–29]. It is important to recognize that the material response observed in the laboratory is a result of many different micro-mechanical processes, such as mineral particle rolling and sliding in granular soils, micro-cracking in brittle rocks, and mineral particle rotation and translation in the cement matrix of soft rocks. Ideally, any localization model for geomaterials must represent all of these processes. However, current limitations of experimental and mathematical modeling techniques in capturing the evolution in the micro-scale throughout testing have inhibited the use of a micro-mechanical description of the localized deformation behavior.

To circumvent the problems associated with the micro-mechanical modeling approach, a macro-mechanical approach is often used. For soils, this approach pertains to the specimen being considered as a macro-scale element from which the material response may be inferred. The underlying assumption is that the specimen is prepared uniformly and deformed

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homogeneously enough to allow extraction of the material response from the specimen response. However, it is well known that each specimen is unique, and that two identically prepared samples could exhibit different mechanical responses in the regime of instability even if they had been subjected to the same initially homogeneous deformation field. This implies that the size of a specimen is too large to accurately resolve the macro-scale field, and that it can only capture the strain localization phenomena in a very approximate way.

In this paper, we adopt a more refined approach to investigating strain localization phenomena based on a meso-scale description of the granular material behavior. As a matter of terminology, the term "meso-scale" is used in this paper to refer to a scale larger than the grain scale (particle scale) but smaller than the element, or specimen, scale (macro-scale). This approach is motivated primarily by the current advances in laboratory testing capabilities that allow accurate measurements of material imperfection in the specimens, such as X-ray computed tomography (CT) and digital image processing (DIP) in granular soils [1,4,8,9,29]. For example, Fig. 1 shows the result of a CT scan on a biaxial specimen of pure silica sand having a mean grain diameter of 0.5 mm and prepared via air pluviation. The gray level variations in the image indicate differences in the meso-scale local density, with lighter colors indicating regions of higher density (the large white spot in the lower level of the specimen is a piece of gravel). This advanced technology in laboratory testing, combined with DIP to quantitatively transfer the CT results as input into a numerical model, enhances an accurate meso-scale description of granular material behavior and motivates the development of robust meso-scale modeling approaches for replicating the shear banding processes in discrete granular materials.

The modeling approach pursued in this paper utilizes nonlinear continuum mechanics and the finite element method, in combination with a constitutive model based on critical state plasticity that captures both hardening and softening responses depending on the state of the material at yield. The first plasticity model exhibiting such features that comes to mind is the classical modified Cam-Clay [24,30–36]. However, this model may not be robust enough to reproduce the shear banding processes, particularly in sands, since it was originally developed to reproduce the hardening response of soils on the "wet" side of the critical state line, and not the dilative response on the "dry" side where this model poorly replicates the softening behavior necessary to trigger strain localization. To model the strain localization process more accurately, we use an alternative critical state formulation that contains an additional constitutive variable, namely, the state parameter ψ [37–39]. This parameter determines whether the state point lies below or above the critical state line, as well as enables a complete "detachment" of the yield surface from this line. By "detachment" we mean that the initial position of the critical state line and the state of stress alone do not determine the density of the material. Instead, one needs to specify the spatial variation of void ratio (or specific volume) *in addition* to the state parameters required by the classical Cam-Clay models. Through the state parameter ψ we can now prescribe quantitatively any measured specimen imperfection in the form of initial spatial density variation.

Specifically, we use classical plasticity theory along with a variant of "Nor-Sand" model proposed by Jefferies [38] to describe the constitutive law at the meso-scale level. The main difference between this and the classical Cam-Clay model lies in the description of the evolution of the plastic modulus. In classical Cam-Clay model the character of the plastic modulus depends on the sign of the plastic volumetric strain increment (determined from the flow rule), i.e., it is positive under compaction (hardening), negative under dilation (softening), and is zero at critical state (perfect plasticity). In sandy soils this may not be an accurate representation of hardening/softening responses since a dense sand could exhibit an initially contractive behavior, followed by a dilative behavior, when sheared. This important feature, called phase transformation in the literature [40,41], cannot be reproduced by classical Cam-Clay models. In the present formulation the growth or collapse of the yield surface is determined by the deviatoric component of the plastic strain increment and by the position of

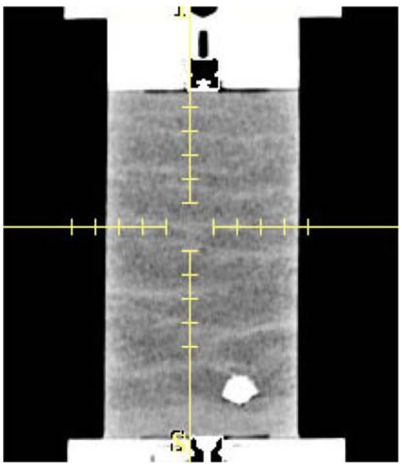

Fig. 1. Cross-section through a biaxial test specimen of silica sand analyzed by X-ray computed tomography; white spot is a piece of gravel.