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Particle and Kalman filtering for state estimation and control of DC motors

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ABSTRACT

State estimation is a major problem in industrial systems. To this end, Gaussian and nonparametric filters have been developed. In this paper the Kalman Filter, which assumes Gaussian measurement noise, is compared to the Particle Filter, which does not make any assumption on the measurement noise distribution. As a case study the estimation of the state vector of a DC motor is used. The reconstructed state vector is used in a feedback control loop to generate the control input of the DC motor. In simulation tests it was observed that for a large number of particles the Particle Filter could succeed in accurately estimating the motor's state vector, but at the same time it required higher computational effort.

1. Introduction

During recent years, there has been significant effort in improving the performance of electric motors. The applications of AC motors are mainly concerned with motion transmission systems [1]. On the other hand, DC motors are widely used in industrial systems, such as robotic manipulators, because their control is relatively simple and they are reliable for a wide range of operating conditions. DC motors are usually modelled as linear systems and then linear control approaches are implemented [2]. Additionally, controllers for nonlinear DC motor models have been developed [3]. If the nonlinearities of the motor are known functions, then adaptive tracking control methods with the technique of input–output linearization can be used [4–7]. When these nonlinearities or disturbances are unknown, neural or fuzzy control can be more suitable for ensuring the satisfactory performance of the closed-loop system [8–15].

The possibility of reducing the number of sensors involved in the control loops of electric motors has been also studied. To this end, feedback control with observer-based state estimation, is also of interest for research in the areas of power electronics and control. In [16] state vector estimation with the use of a high-gain observer and adaptive control is applied to a transverse flux permanent magnet motor. In [17] maximum likelihood hypothesis testing is used to derive a nonlinear observer for estimating the state vector of dynamical systems. The proposed observer is applied to friction estimation and diagnosis in a rotating machine. In [18] Kalman filtering is proposed for estimating the state vectors of a discrete-time nonlinear system. The estimated state vector generates residuals which in turn are used by a fault diagnosis algorithm which decides on the system's condition. In [19,20] observer-based feedback control is proposed for linear systems.

It is known that for linear systems subject to Gaussian measurement or process noise the Kalman Filter is the optimal state estimator, since it results in minimization of the trace of the estimation error's covariance matrix [21,22]. For nonlinear systems, subject to Gaussian noise one could use the generalization of the Kalman Filter as formulated in terms of the Extended Kalman Filter. The Extended Kalman Filter is based on a linearization of the system dynamics using a first order Taylor expansion, and thus there is neither a proof of its convergence, nor a proof that the estimation produced by the EKF satisfies optimality criteria. To overcome the limitations of KF and of EKF the Particle Filter (PF) has been proposed [23–25]. It has been shown that PF-based state estimation is suitable for industrial systems, subject to non-Gaussian noise, such as the CSTR system (continuously stirred reactor), and the four-tank system [26]. Moreover, PF-based state estimation has been proposed for control and fault diagnosis tasks in mechanical/robotic systems [27–32]. The particle filtering algorithm reminds one of the genetic algorithms where a number of N particles is subject to a mutation mechanism which corresponds to the prediction stage, and to selection mechanism which corresponds to the correction stage [33–35].

The main features of particle filtering are summarized in the following: (i) it is a nonparametric state estimator since it is not dependant on assumptions about the p.d.f. of the process and measurement noises and can function equally well for

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Gaussian and non-Gaussian noise distributions. (ii) it has improved performance over the Extended Kalman Filter, since it can provide optimal estimation in non-Gaussian state-space models, as well as in estimation of linear and nonlinear models [36–40], (iii) it is not based on any linearization of the system dynamics and can be very efficient in state estimation in nonlinear dynamical systems, (iv) when applied to linear systems with Gaussian noise, the Particle Filter asymptotically approaches the Kalman filter if the number of particles becomes large.

The structure of the paper is as follows: In Section 2 the problem of state estimation and control of DC motors is analyzed. In Section 3 state estimation with the use of Kalman Filtering is discussed. In Section 4, the particle filtering algorithm for state estimation in dynamical systems is introduced. The prediction and correction stages are explained. In Section 5 issues for improved resampling and substitution of the degenerate particles in the Particle Filter algorithm are discussed. In Section 6 simulation experiments are carried out to evaluate the performance of the Kalman Filter and the Particle Filter in reconstructing the state of the DC motor and subsequently in using this state estimation in feedback control. Finally, in Section 7 concluding remarks are stated.

2. State estimation in the control loop of DC motors

2.1. DC motor modelling

A direct current (DC) motor model converts electrical energy into mechanical energy. The torque developed by the motor shaft is proportional to the magnetic flux in the stator field and to the current in the motor armature (iron cored rotor wound with wire coils). There are two main ways of controlling a DC motor: the first one named *armature control* consists of maintaining the stator magnetic flux constant, and varying (use as control input) the armature current. Its main advantage is a good torque at high speeds and its disadvantage is high energy losses. The second way is called *field control*, and has a constant voltage to set up the armature current, while a variable voltage applied to the stator induces a variable magnetic flux. Its advantages are energy efficiency, inexpensive controllers and its disadvantages are a torque that decreases at high speeds [41]. The position (angle) of the motor is measured using an encoder, while the motor's rotational speed and angular acceleration can be also recorded by the use of a tachometer and an accelerometer respectively.

The objective is to estimate the state vector of the DC motor from encoder measurements using either the Kalman Filter or the Particle Filter, and to use subsequently the reconstructed state vector in a feedback control law. The motor's angle has to follow accurately a specified trajectory. A model that approximates the dynamics of the DC motor is derived as follows: the torque developed by the motor is proportional to the stator's flux and to the armature's current thus one has

$$\Gamma = k_f \Psi K_\alpha I \quad (1)$$

where Γ is the shaft torque, Ψ is the magnetic flux in the stator field, and I is the current in the motor armature. Parameter $K_\alpha = PZ_\alpha/2\pi\alpha$ is related to the number of poles P, the number of wires in each armature winding Z_α and the number of parallel windings α. Parameter k_f is related to the magnetic flux and may vary with the length of the air-gap between the stator and the rotor as well as with the magnetic permeability. Since the flux is maintained constant the torque of Eq. (1) can be written as

$$\Gamma = k_T I, \quad \text{where } k_T = k_f \Psi K_\alpha. \quad (2)$$

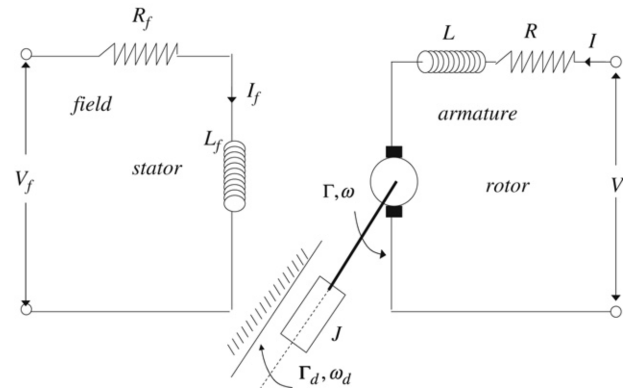

Fig. 1. Parameters of the DC motor model.

Apart from this, when a current carrying conductor passes through a magnetic field, a voltage V_b appears corresponding to what is called the electromagnetic force (EMF)

$$V_b = k_e \omega \quad (3)$$

where ω is the rotation speed of the motor shaft. The constants k_T and k_e have the same value. Kirchhoff's law yields the equation of the motor (Fig. 1):

$$V - V_{\text{res}} - V_{\text{coil}} - V_b = 0 \quad (4)$$

where V is the input voltage, $V_{\text{res}} = RI$ is the armature resistor voltage (R denotes the armature resistor) and $V_{\text{coil}} = L\dot{I}$ is the armature inductance voltage. The motor's electric equation is then

$$L\dot{I} = -k_e\omega - RI + V. \quad (5)$$

From the mechanics of rotation it holds that

$$J\dot{\omega} = \Gamma - \Gamma_{\text{damp}} - \Gamma_d. \quad (6)$$

The DC motor model is finally

$$\begin{aligned} L\dot{I} &= -k_e\omega - RI + V \\ J\dot{\omega} &= k_e I - k_d\omega - \Gamma_d \end{aligned} \quad (7)$$

with the following notations

Notation	Significance
L	Armature inductance
I	Armature current
k_e	Motor electrical constant
R	Armature resistance
V	Input voltage, taken as control input
J	Motor inertia
ω	Rotor rotation speed
k_d	Mechanical dumping constant
Γ_d	Disturbance torque

where the armature designates the iron cored rotor wound with wired coils. Assuming $\dot{\Gamma}_d = 0$ and denoting the state vector as $[x_1, x_2, x_3]^{\mathrm{T}} = [\theta, \dot{\theta}, \ddot{\theta}]^{\mathrm{T}}$, a linear model of the DC motor is obtained:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \dfrac{-k_e^2 - k_d R}{JL} & \dfrac{-JR - k_d L}{JL} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dfrac{k_e}{JL} \end{pmatrix} V. \quad (8)$$

Usually the DC-motor model is considered to be linear by neglecting the effect of armature reaction or by assuming that the compensating windings remove this effect. Introducing the