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3D Crack propagation in unreinforced concrete. A two-step algorithm for tracking 3D crack paths

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Abstract

Tensile failure of unreinforced concrete involves progressive micro-cracking, and the related strain-softening can coalesce into geometrical discontinuities, which separate the material. Advanced mechanical theories and numerical schemes are required to efficiently and adequately represent crack propagation in 3D. In this paper we use the concept of strong discontinuities to model concrete failure. We introduce a cohesive fracture process zone, which is characterized by a transversely isotropic traction–separation law. We combine the cohesive crack concept with the partition of unity finite element method, where the finite element space is enhanced by the Heaviside function. The concept is implemented for tetrahedral elements and the failure initialization is based on the simple (non-local) Rankine criterion. For each element we assume the embedded discontinuity to be flat in the reference configuration, which leads to a non-smooth crack surfaces approximation in 3D, in general; different concepts for tracking non-planar cracks in 3D are reviewed. In addition, we propose a two-step algorithm for tracking the crack path, where a predictor step defines discontinuities according to the (non-local) failure criterion and a corrector step draws in non-local information of the existing discontinuities in order to predict a 'closed' 3D crack surface; implementation details are provided. The proposed framework is used to analyze the predictability of concrete failure by two benchmark examples, i.e. the *Nooru-Mohamed test*, and the *Brokenshire test*. We compare our numerical results, which are mesh independent, with experimental data and numerical results adopted from the literature.

Keywords: 3D crack propagation; Unreinforced concrete; Tracking algorithm; PUFEM

1. Introduction

Prevention of failure by fracture of structural components in service is a major concern in engineering (see, e.g., [1]). In particular, the study of structural failure of concrete has been of extensive academic and industrial importance in the past three decades.

At least since the findings in [20] we know that (classical) linear fracture mechanics of sharp cracks is an inadequate concept to be used for concrete structures, an assertion which has been supported by several other authors in the meanwhile. Tensile failure of unreinforced concrete involves progressive micro-cracking, debounding and other complex irreversible processes of internal damage. The associated strain-softening can coalesce into a geometrical discontinuity, which separates the material. Hence, the discrete crack concept is the approach that reflects this type of phenomena closest.

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Concrete may be considered as a quasi-brittle material, in which a sizeable nonlinear zone develops at the fracture front, which is almost entirely filled by the fracture process zone, and wherein plasticity effects are negligible [5]. Concrete failure shows pronounced strain-softening phenomena, and hence a description within polar (local) continuum mechanics fails (energy dissipated at failure is incorrectly predicted to zero [6]) and advanced theories are required.

Constitutive modeling of concrete failure has not yet resulted in the model, or the set of models, capable of representing the whole range of mechanical characteristics of concrete in a consistent and robust manner. Numerous constitutive models for concrete have been proposed in the past which are based on plasticity theories, fracture theories (fixed crack, rotating crack and multi-non-orthogonal fixed crack methods), damage theories and formulations which couple these approaches, see [16,17]. For comparative studies of 3D-constitutive models for concrete pointing out the diverging results, see, for example, [40].

One objective of the present work is the constitutive modeling of 3D crack propagation in unreinforced concrete for mixed mode situations, where, for a first approximation, fracture parameters for the opening mode (mode I) are used. This holds if the amount of shear stresses compared with tensile stresses is moderate, and hence shear friction and aggregate interlocking are negligible. In Section 2 we briefly review the needed continuum mechanical framework, the kinematics to capture concrete failure by means of strong discontinuities, the finite element formulation, and follow in Section 3 the pioneering works [11,4] for elasto-plastic fracture in metals, and [14] for quasi-brittle failure of concrete materials. We introduce a cohesive fracture process zone and employ a recently proposed transversely isotropic cohesive model [13], which is based on the theory of invariants [48]. It is suitable to describe the typical traction–separation behavior of tensile failure in concrete in a phenomenological way. For additional particularizations of Traction Separation Laws (TSLs), for example, for cubic, exponential and trilinear TSLs see [27,28,52]. More references utilizing the idea of cohesive zones to model material failure, and a general discussion on the limitations of cohesive models are provided in the seminal work [5]. Note that, alternatively, concrete failure may also be captured by means of weak discontinuities, where a jump is added to the strain field [37,7,24,47]. In [33] it is shown that a strong discontinuity problem may be regarded as a limiting case of a weak discontinuity problem as the width of the localization band tends to zero.

Another objective of the present work is the numerical modeling of 3D crack propagation in unreinforced concrete by employing the Partition of Unity Finite Element Method (PUFEM), which allows a treatment of the separation of a material in a very robust and efficient way (for details on the PUFEM see [25]). We enhance the finite element space by the Heaviside function according to [56], and combine the method with the cohesive crack concept. This leads to a finite element formulation with embedded strong discontinuities, which has several advantages over traditional smeared and discrete approaches (see, for example, [19,23]). It is worth noting that, alternatively, strong discontinuities have been combined with finite element formulations, which are based on the mixed Enhanced Assumed Strain (EAS) method, as proposed in [45]. EAS-methods for the geometrically linear regime are presented in [44,2,31,55], while [3,49] extend the concept to the nonlinear regime. A comparative study of the formulations based on the EAS concept was recently presented in the authors' work [12]. Therein we concluded that either the three-field Hu–Washizu variational formulation or the kinematics of strong discontinuities are satisfied, but not both together, which may lead to not meaningful numerical results for the separation process governed by, for example, stress locking phenomena.

The fully 3D PUFEM implementation is based on two variational statements arising from a standard single-field variational formulation in spatial description. The numerical model has been utilized for tetrahedral elements using the multi-purpose finite element analysis program FEAP [51]. A critical task for applying the PUFEM to 3D is the representation of the crack surface. We assume that the embedded discontinuity within a particular finite element is a flat surface in the reference configuration (for details see [13]), therefore, the proposed concept leads, in general, to a non-smooth crack surface in 3D. In Section 4 we discuss different concepts for representing non-planar crack surfaces in 3D documented in the literature. Perhaps the most popular approaches is the Level Set Method (LSM) [50,26] (originally proposed in [38]), and a global tracking algorithm which is based on the solution of a kind of 'heat conduction' problem [35]. Recently, however, the study in [54] claims that the standard LSM is not an ideal solution for characterizing cracks, while the algorithm proposed in [35] requires the solution of a 'thermal-like' problem before each mechanical loading step, which is related to high computational cost. Hence, in Section 5 we describe in detail a two-step algorithm for tracking multiple non-planar cracks in 3D, which is fundamentally different from the above mentioned approaches. The first step, the predictor say, can be seen as a generalization of a (simple) local strategy of tracking 2D strong discontinuities, documented in [55], to 3D. The second step, the corrector say, draws in non-local information of the existing discontinuities in order to predict a 'closed' 3D crack surface. The proposed non-local smoothing strategy effectively circumvents topological difficulties, which may arise from the predictor step. Details about the implementation of the algorithm for tracking the 3D crack path are provided. The proposed computational framework, which combines PUFEM with the new smoothing algorithm for non-planar 3D cracks, is now used to analyze the predictability of concrete failure by means of two representative numerical examples, i.e. the *Nooru-Mohamed test*, a mixed mode failure test, and the *Brokenshire test*, a torsion failure test. The objective is to present the robustness and efficiency of the proposed framework. The computational results are documented in Section 6, and compared with experimental and numerical data adopted from the literature.