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Original

A gradient descent control for output tracking of a class of non-minimum phase nonlinear systems

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Abstract

In this paper we present a new approach to design the input control to track the output of a non-minimum phase nonlinear system. Therefore, a cascade control scheme that combines input–output feedback linearization and gradient descent control method is proposed. Therein, input–output feedback linearization forms the inner loop that compensates the nonlinearities in the input–output behavior, and gradient descent control forms the outer loop that is used to stabilize the internal dynamics. Exponential stability of the cascade-control scheme is provided using singular perturbation theory. Finally, numerical simulation results are presented to illustrate the effectiveness of the proposed cascade control scheme.

Keywords: Input–output feedback linearization; Non-minimum phase system; Singular perturbed system; Gradient descent control

1. Introduction

The control of nonlinear non-minimum phase systems is a challenging problem in control theory and has been an active research area for the last few decades. This technique, as a matter of fact, was successfully established in various practical applications (Bahrami, Ebrahimi, & Asadi, 2013; Cannon, Bacic, & Kouvaritakis, 2006; Charfeddine, Jouili, Jerbi, & Benhadj Braiek, 2010; Jouili & BenHadj, 2015; Sun, Li, Gao, Yang, & Zhao, 2016). This system control is a delicate task owing to the fact that it is a nonlinear system with non-minimum phase, and that it is also characterized by a dynamic prone to the instability of the dynamics of zero (Jouili & Jerbi, 2009; Jouili, Jerbi, & Benhadj Braiek, 2010; Kazantzis, 2004; Naiborhu, Firman, & Mu'tamar, 2013). In fact there exist no generic methods for controller synthesis and design (Khalil, 2002). Several fundamental methods in the output tracking problems on nonlinear non-minimum phase systems have been proposed in this area.

Hirschorn and Davis (1998), Isidori (1995), and Hu et al. (2015) have proposed the stable inversion method to the tracking problem with unstable zero dynamics. This method tries to find a stable solution for the full state space trajectory by steering from the unstable zero dynamics manifold to the stable zero dynamics manifold.

Khalil (2002) has derived a minimum phase approximation to a single-input single-output nonlinear, non-minimum phase system. An input–output linearizing controller is designed for this approximation and then applied to the non-minimum phase plant. This leads to a system that is internally stable. Naiborhu and Shimizu (2000) presented a controller designed based upon an internal equilibrium manifold where this controller pushes the state of a nonlinear non-minimum phase system toward that manifold. This has afforded approximate output tracking for nonlinear non-minimum phase systems while maintaining internal stability.

Kravaris and Soroush have developed several results on the approximate linearization of non minimum phase systems (Kanter, Soroush, & Seider, 2001; Kravaris & Daoutidis, 1992; Kravaris, Daoutidis, & Wright, 1994; Soroush & Kravaris, 1996). For instance Kanter et al. (2001) and Kravaris et al. (1994) investigated the system output which is differentiated as many times as the order of the system where the input derivatives

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Nomenclature

x	vector of state variables
u	control input
y	output variable
ξ	vector of slow state variables
η	vector of fast state variables of the internal dynamics
u^*	local minimal point of an control variable u
y	scalar output
y_{ref}	reference trajectory for the output
Z	state vector of reduced subsystem
η_{ref}	virtual desired output
u_{QSS}	QSS control input
u_{ar}	artificial input
$V(x)$	Lyapunov function
$\Upsilon(u)$	performance function of an control variable u
$\psi(Z)$	descent function

that appear in the control law are set to zero when computing the state feedback input. Bortoff (1997) has studied the system input–output feedback of the first linearized. Then, the zero dynamics is factorized into stable and unstable parts. The unstable part is approximately linear and independent of the coordinates of the stable part. Charfeddine, Jouili, and Benhadj Braiek (2015) dismissed a part of the system dynamics in order to make the approximate system input-state feedback linearizable. The neglected part is then considered as a perturbation part that vanishes at the origin. Next, a linear controller is designed to control the approximate system.

Moreover, an original technique of control based on an approximation of the method of exact input–output linearization, was proposed in the works (Charfeddine, Jouili, Jerbi, & Benhadj Braiek, 2011; Guardabassi & Savaresi, 2001; Guemghar, Srinivasan, Mullhaupt, & Bonvin, 2002; Hauser, Sastry, & Kokotovic, 1992). The approximation (Charfeddine et al., 2011) is used to improve the desired control performance. A cascade control scheme has been considered (Charfeddine, Jouili, & Benhadj Braiek, 2014; Yakoub, Charfeddine, Jouili, & Benhadj Braiek, 2013) that combines the input–output feedback linearization and the backstepping approach.

On the other hand, Firman, Naiborhu, and Saragih (2015) have applied the modified steepest descent control for that system output will be redefined such that the system becomes minimum phase with respect to a new output.

In this paper, we address the problem of tracking control of a single-input single-output of non-minimum phase nonlinear systems. The idea here is to transform the given system into Byrnes–Isidori normal form, then to use the singular perturbed theory in which a time-scale separation is artificially introduced through the use of a state feedback with a high-gain for the linearized part. The gradient descent control method (Naiborhu & Shimizu, 2000) is introduced to generate a reference trajectory for stabilizing the internal dynamics.

This results in a cascade control scheme, where the outer loop consists of a gradient descent control of the internal dynamics, and the inner loop is the input–output feedback linearization.

The stability analysis of the cascade control scheme is provided using results of singular-perturbation theory (Khalil, 2002).

The rest of this paper is organized as follows. In Section 2, some mathematical preliminaries are presented. The proposed cascade control scheme and the stability analysis are given in Sections 3 and 4, respectively. In Section 5, the effectiveness of the proposed control scheme is illustrated by numerical examples. Finally, this paper will be closed by a conclusion and a future works presentation.

2. Theoretical background

In this paper, we consider a single-input single-output nonlinear system of the form:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \tag{1}$$

where $x \in \Re^n$ is the n-dimensional state variables, $u \in \Re$ is a scalar manipulate input and $y \in \Re$ is a scalar output. $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are smooth functions describing the system dynamics.

2.1. Exact input–output feedback linearization

The input output linearization is based on two concepts: the concept of relative degree and the concept of state transformation.

The relative degree r of the system (1) is defined as the number of derivation of the output y needed to appear in the input u, such as $\forall x \in \Re^n$:

$$\begin{cases} L_f^k h(x) = 0 \,\forall \quad 1 \leq k \leq r-1 \\ L_g L_f^{(r-1)} h(x) \neq 0 \end{cases} \tag{2}$$

If $r \leq n$, then system (1) can be feedback linearized into Byrnes–Isidori normal form (Isidori, 1995) using the following steps:

Step 1: We apply the following control law

$$u(x) = \frac{v - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \tag{3}$$

with $v = y^{(r)}$

This control law compensates the nonlinearities in the input–output behavior.

Step 2: First, system (1) is transformed into normal form (Isidori, 1995) through a nonlinear change of coordinates: