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Geometry dependence of 2-dimensional space-charge-limited currents

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ABSTRACT

The space-charge-limited current in a zero thickness planar thin film depends on the geometry of the electrodes. We present a theory which is to a large extent analytical and applicable to many different lay-outs. We show that a space-charge-limited current can only be sustained if the emitting electrode induces a singularity in the field and if the singularity induced by the collecting electrode is not too strong. For those lay-outs where no space-charge-limited current can be sustained for a zero thickness film, the real thickness of the film must be taken into account using a numerical model.

1. Introduction

When charge carriers are injected into an electrically poorly conducting medium, the current is space-charge-limited and when the medium has Ohmic conductivity, with increasing voltage, the current eventually becomes also space-charge-limited. This phenomenon has been known for a long time in a one-dimensional (1D) setting as described by the Mott-Gurney equation [1].

$$J = \frac{9}{8}\mu\varepsilon\frac{V^2}{L^3} \tag{1}$$

with V the voltage, J the current density, L the width of the insulator, ε it's dielectric constant and μ the mobility of the carriers. Eq. (1) holds in particular for single carrier injection under perfect injection conditions, meaning that the electric field is zero at the injecting electrode. Similar behavior has been observed in a planar two-dimensional (2D) setting in organic thin films [2–5] and more recently in several types of monolayers [6,7]. In Ref. [8] we derived the following 2D version of Eq. (1) for an infinitesimally thin layer between two semi-infinite co-planar electrodes

$$K = \frac{2}{\pi}\varepsilon\mu\frac{V^2}{L^2} \tag{2}$$

where K is the surface current density, and similar additional results were also obtained for a photoconductor. Subsequently we discovered a paper by Grinberg et al. [9] where besides this “strip” lay-out two more lay-outs were considered: a thin film between two parallel electrodes perpendicular to the film (“plane” lay-out) and a thin film with small “edge” electrodes. These lay-outs are shown in Fig. 1 together with the idealized models used to calculate the current. Indeed only “the limiting case of a vanishing film thickness” was considered and the relevant equations were solved numerically with the aim to obtain the prefactor α occurring in the general expression

$$K = \alpha\varepsilon\mu\frac{V^2}{L^2} \tag{3}$$

They found respectively $\alpha_{strip} \approx 0.7$, $\alpha_{plane} \approx 1$ and $\alpha_{edge} \approx 0.57$. When we applied our analytical method to these idealized “plane” and “edge” models we found that actually $\alpha_{plane}=\alpha_{edge}=0$, meaning that in these idealized structures no space-charge-limited (SCL) current can be sustained and to obtain a practical result the film thickness must be taken into account.

In this paper we explore the dependence of the prefactor α in (3) on the lay-out systematically and analytically as much as possible.

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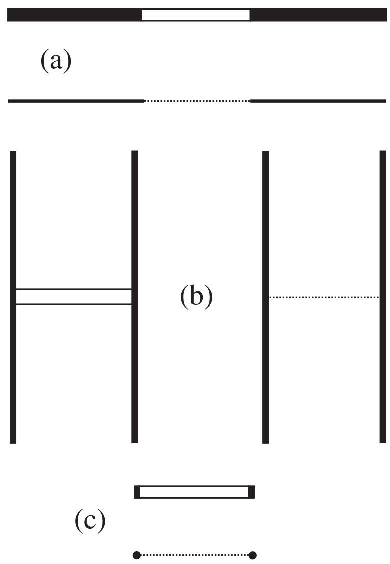

Fig. 1. Different 2D thin film lay-outs considered by Grinberg et al. [9]: (a) "strip" lay-out, (b) "plane" lay-out and (c) "edge" lay-out. For each lay-out the actual lay-out with a non-zero film thickness is shown next to the idealized one with a zero thickness thin film and which is used in their model.

In Section 2 we explain our method by deriving the value of α_{strip} for the reference case of two semi-infinite coplanar electrodes. In Section 3 this result is extended to other lay-outs using conformal transformations and as a result we obtain several limits leading to the zero result for the "plane" lay-out. In Section 4 we consider an approximate and numerical model for a thin film between planar electrodes but with a non-zero thickness. In Section 5 we turn our attention to electrodes with finite width, in particular the idealized "edge" electrodes. In this case a slightly different method must be used and a single numerical integration is required to find α. In the last Section 6 we consider asymmetrical lay-outs.

In their paper Grinberg et al. refer to a paper by Geurst [10] where the exact expression $\frac{2}{\pi}$ for the prefactor occurring in (2) for the "strip" lay-out was derived, as far as we know, for the first time. This result was found by solving analytically a boundary value problem for the square of the complex electric field. In our method [8] the problem is reduced to solving a non-linear integral equation with a known solution, which was published by Peters [11]. We will also show how these two methods are related. A totally different approach to the problem, based on E-Infinity theory, was published by Zmeskal et al. [12].

Eqs. (1)–(3) and the rest of this paper holds for drift transport. For ballistic transport Eq. (1) must be replaced by the (1D) *Child-Langmuir* law. Some studies of 2D versions of the *Child-Langmuir* law have been published for parallel electrodes [13–17]. In what follows we consider the injection of positive charges from the anode (emitter) to the cathode (collector) but the results are obviously equally valid for negative charges.

2. Semi-infinite coplanar electrodes

Photoconductors are often contacted by two interdigitated electrodes and if the fingers are much wider than the gaps in between then this lay-out can be approximated well by two semi-infinite coplanar electrodes as shown in Fig. 2.

In the calculations we will use only normalized quantities with the channel width L=2 and the applied voltage V=1. The true surface current density K is then written as

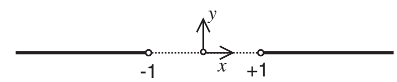

Fig. 2. Idealized "strip" lay-out for 2D SCL current flow. The electrodes are shown as thick lines and the actual channel where current flows by the broken line. The small circles have no physical meaning and are used to mark specific points only. We use coordinates (x,y) as indicated with the complex variable z=x+jy.

$$K = 2\varepsilon\mu\frac{\rho}{2\varepsilon}E_x\frac{4V^2}{L^2} \tag{4}$$

where the in-plane component of the electric field E_x and the upper out-of-plane component $E_y^+ = \frac{\rho}{2\varepsilon}$, with ρ the surface charge density, are normalized by $2V/L$. Comparing with (3) we then find the prefactor from the equation

$$\alpha = 8E_y^+E_x \tag{5}$$

where the field components must still satisfy Maxwell's equations. Assuming E_y^+ known for all x, and using the Green's function, E_x is easily found as

$$E_x(x) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{E_y^+(t)}{x-t}\mathrm{d}t \tag{6}$$

where the integral is a Cauchy principal value integral. From this equation we learn that both field components are connected by a *Hilbert*-transform over the real axis. Since the *Hilbert*-transform equals it's own inverse, except for a sign reversal, we find immediately

$$E_y^+(x) = \frac{1}{\pi}\int_{-1}^{+1}\frac{E_x(t)}{t-x}\mathrm{d}t \tag{7}$$

where we also used the boundary condition that along the electrodes E_x=0. Substituting (7) in (5) we find that the unknown function ϕ=E_x most be chosen in such a way that the following expression

$$\alpha = \frac{8}{\pi}\phi(x)\int_{-1}^{+1}\frac{\phi(t)}{t-x}\mathrm{d}t \tag{8}$$

is a constant within the gap -1<x<1 and zero elsewhere. This type of equation can be solved analytically [8,11] but to obtain α the explicit solution is not needed (in Section 5 we explain how the field components can be obtained). It suffices to integrate (8) over the gap after removing possible singularities. This condition is necessary for reversing the order of integration in the rhs.[1] In this particular case the in-plane component of the electric field shows a singularity near x=1 only, whereas $E_x(-1)$=0 because of the perfect injection boundary condition. Multiplying Eq. (8) with the factor (1−x) and integrating we obtain

$$\alpha = \frac{4}{\pi}\int_{-1}^{+1}\phi(x)(1-x)dx\int_{-1}^{+1}\frac{\phi(t)}{t-x}\mathrm{d}t \tag{9}$$

[1] Formally $\int_a^b\phi_1(x)\mathrm{d}x\int_a^b\frac{\phi_2(t)}{t-x}\mathrm{d}t = \int_a^b\phi_2(t)\mathrm{d}t\int_a^b\frac{\phi_2(x)}{t-x}\mathrm{d}x$ if $\phi_1 \in L_{p_1}$, $\phi_2 \in L_{p_2}$ with $p_1^{-1}+p_2^{-1} \le 1$ [18]. Since p_2<2 we need p_1>2.