Journal of Electrostatics 87 (2017) 19–25

Contents lists available at ScienceDirect

Journal of Electrostatics

journal homepage: www.elsevier.com/locate/elstat

Static dielectric constant assessment from capacitance over a wide range of electrode separations

Gokul Raj R, C.V. Krishnamurthy*

Measurement and Modeling Lab, Department of Physics, Indian Institute of Technology Madras, Chennai 600036, India

A R T I C L E I N F O

Article history:
Received 21 November 2016
Received in revised form
28 February 2017
Accepted 6 March 2017

Keywords:
Circular disc
Capacitance
Dielectric constant
Fringing fields
Parallel electrode capacitor

A B S T R A C T

Static dielectric constant extraction from two-electrode capacitance measurement over a wide range of electrode separations and dielectric constants involves careful assessment of fringe fields. Finite-element method has been employed to compute capacitance and quantify fringe fields for parallel electrode capacitor of (finite thickness, radii r, electrode separation d), with a homogeneous dielectric medium extending up to the geometric limits of the electrodes. Two distinct regimes, in the fringe field contributions are seen. A procedure to extract the static dielectric constant has been proposed for the first regime and a validation has been provided for the same.

1. Introduction

Static dielectric constant measurements are usually done by forming a simple capacitor with the medium sandwiched between two electrodes. Experimentally measurable quantity is capacitance and one has to use the geometrical parameters to extract dielectric constant information from the experiment. For large electrode area (A), small electrode separation (d) and negligible electrode thickness the approximate expression,

$$C_{ideal} = \frac{\varepsilon_r \varepsilon_0 A}{d} = \varepsilon_r C_0 \tag{1}$$

derived analytically, neglecting fringing effects has been widely employed to determine ε_r the relative permittivity. Here, ε_0 is the free space permittivity. C_0 denotes the ideal capacitance for relative permittivity $\varepsilon_r = 1$. Capacitance measurements for the parallel circular electrode configuration that deviate from the ideal capacitance are usually handled through stray capacitance and fringe field capacitance corrections.

Attempts to deal with the capacitance due to fringe fields have been of two kinds.

1) Minimize fringe fields through use of guard ring.
2) Evaluate fringe field capacitance through the solution of Laplace's equation as a small correction term using analytical, semi-analytical and numerical techniques.

Table 1 provides a summary of various approaches, reported in the literature, to evaluate/measure capacitance for parallel circular electrodes arranged in various configurations as shown in Fig. 1.

Kirchhoff [4,5] deduced an approximate formula to quantify fringe fields for parallel electrode capacitor (no dielectric medium) at small electrode separations. Ali Naini and Mark Green [7] tried to eliminate the logarithmic divergence of fringing field effect in Kirchoff's formula by incorporating thickness for circular electrodes. Experiments were performed, without any dielectric medium, for aspect ratio range $0.01 \le \frac{d}{r} \le 0.5$. The thickness-correction formula was found to work well for $\frac{d}{r} < 0.1$ but failed above it. Carlson and Illman [10] provided a numerical evaluation of solutions to Love's equation and obtained capacitance for various aspect ratios $10^{-3} \le \frac{d}{r} \le 10$ with $\varepsilon_r = 1$. They have also compared their numerical values of capacitance with previous investigators [6,8]. Wintle and Goad [9] employed semi-analytical methods for finding capacitances of circular disk capacitors having different dielectric constants for aspect ratios $0.01 \le \frac{d}{r} \le 10$. They compared excess capacitance with Kirchhoff equation for $\frac{d}{r} < 0.1$ with $\varepsilon_r = 1$. A recent publication [1] reviews analytical and semi-

---

* Corresponding author.
E-mail address: cvkm@iitm.ac.in (C.V. Krishnamurthy).

http://dx.doi.org/10.1016/j.elstat.2017.03.001
0304-3886/

Table 1
A summary of analytical, semi-analytical and numerical approaches for capacitance assessment.

S.No.	Aspect ratio d/r	Relative permittivity (ε_r)	Plate thickness (h)	Approach	Ref.	Configurations
Circular Electrodes						
1.	0.007–0.2	1, and 2.02 to 64.9 (liq.)	Not Specified	Experimental	[3]	Fig. 1(a)
2.	0.01–0.5	1	2.3 mm	Experimental	[7]	Fig. 1(a)
3.	0.001–10	1	Negligible	Semi-Analytical	[10]	Fig. 1(a)
4.	0.0001–1	1	Negligible	Semi-Analytical	[8]	Fig. 1(a)
5.	0.01–10	0.5–100	Negligible	Semi-Analytical	[9]	Fig. 1(c)
6.	0.01–10	1	Negligible	Numerical (BEM)	[11]	Fig. 1(a)
7.	0.1–10	1,3	Negligible	Numerical (MoM)	[12]	Fig. 1(c)
8.	0.001–10	1	Negligible	Semi-Analytical	[13]	Fig. 1(a)
9	0.00001–0.01	1	Negligible	Semi-Analytical	[14]	Fig. 1(a)

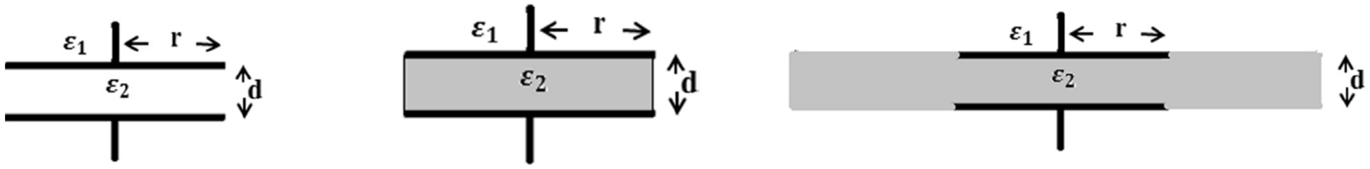

Fig. 1. Different configurations of identical circular disk parallel electrode capacitor (a) same dielectric in every region (b) dielectric extending up to the geometric limit (c) dielectric extending beyond the geometric plate limits.

analytical approaches and presents an analytical treatment for the asymmetric electrode configuration. A procedure to calculate capacitance co-efficient at large aspect ratio of two bodies with different orientations has been reported recently [2].

It can be seen from Table 1 that the most extensively studied case, for the assessment of fringing effects, has been for the relative permittivity $\varepsilon_r = 1$ pertaining to configuration 1(a). Wintle and Goad [9] study configuration 1(c) and tabulate the fringe field capacitance over a wide range of dielectric constants and aspect ratios. It is further seen that effects of electrode thickness are (a) neglected in analytical treatments due to difficulties in formulating the problem statement and (b) largely ignored in experiments since corrective measures are not clear, despite having to employ finite thickness electrodes. More importantly, there has been very little done to study the effect of fringing field capacitance pertaining to configuration 1(b) for $\varepsilon_r > 1$. Practically, schemes to determine relative permittivity, ε_r, from capacitance measurements, using a symmetric electrode configuration (Fig. 1(b)), involving (i) electrodes with finite thicknesses, and (ii) medium to large aspect ratios, do not seem to have been reported.

With the availability of powerful, yet economical, computational resources, numerical approaches, such as those based on FEM, can provide an assessment of simple as well as practical measurement scenarios. The work presented in what follows has been conceived to provide an insight on how fringe fields influence capacitance measurements using a symmetric electrode configuration in the presence of dielectric media and to arrive at a procedure to extract the relative permittivity from capacitance measurements.

Section 1 outlines the FEM approach for the problem and describes the implementational details of the numerical aspects. Section 2 details the FEM-based calculations carried out for a wide range of aspect ratios and relative permittivities to assess the fringe field contributions to capacitance. The results are examined for trends that can be understood and quantified in a useful manner. Section 3 provides an alternative assessment based on total capacitance and establishes trends that can be quantified for practical extraction of relative permittivity. Section 4 presents validation with experimental data.

1.1. Section 1: FEM implementation and results

Finite element method is used to evaluate the capacitance of two parallel circular electrodes with variable spacing and dielectric filling by exploiting the axi-symmetry. The calculations are carried out for a wide range of aspect ratios ($10^{-3} \le \frac{d}{r} \le 10$) and dielectric constants ($1 \le \varepsilon_r \le 80$). Finite element method (FEM) converts the differential equations to difference equations. The discretization of computational domain of interest has been done with triangular meshes. Given the boundary conditions, the potential at different nodes of the meshes are computed which will help in calculating capacitance eventually. Electrostatic module in COMSOL solves the Laplace's equation ($\nabla^2 V = 0$) with zero charge boundary condition/Perfectly insulating boundary conditions, in which the electric field lines are tangential to the boundary ($\hat{n}.\vec{D} = 0$), where $\hat{n}$ is the unit vector normal to the boundary and $\vec{D}$ represents the electric displacement field. The potential difference across the parallel electrodes is provided by specifying one terminal to be at 1 V and the other to be at 0 V. Convergence check on capacitance has been

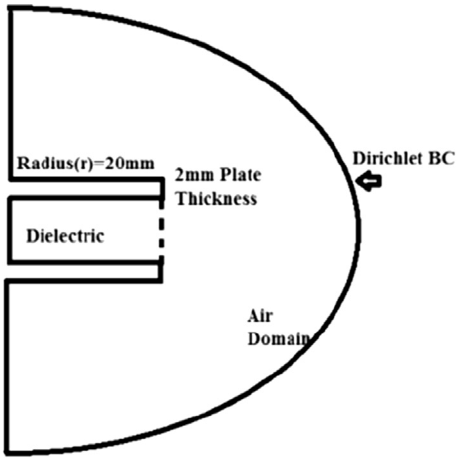

Fig. 2. Schematic representation of parallel electrode capacitor employed in FEM simulation.