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Surface force induced by a point charge in multilayered dielectrics

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A R T I C L E I N F O

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Local surface force density and total force induced by a point charge embedded in a three-layered dielectric system are calculated. The two ratios between the dielectric constants of the three layers are found to play a primary role: they determine the direction of the surface force density and total force, as well as distribution of the surface force density, which can vary monotonically or non-monotonically with the radial position. The position of the charge, however, only affects the magnitude of the forces. The formulation can be extended to establish a theoretical framework for situations involving a distribution of charges.

1. Introduction

Understanding the electrostatic force induced by a distribution of charges in layered dielectric medium is important to many applications, especially in electromechanical devices for semiconductor industry [1–4]. Proper handling of dielectrics in these devices is crucial to the manufacturing of contamination free products. Electrostatic chucks are now widely used to replace mechanical holding systems for semiconducting wafers [5,6]. These chucks normally consist of a planar array of parallel bar electrodes. An attractive electrostatic force is induced with a thinner layer of dielectric between the chuck and the product. For holding purposes, the performance of the electrostatic chucks depends on the magnitude of the generated electrostatic force, which in turn depends on proper selection of the dielectric. Electrostatic forces are also used to induce deformation of thin dielectric films or liquids so that they form patterns, allowing for the technique of electrolithography or electrohydrodynamic lithograph [7,8]. Another interesting application is found in the development of wall climbing robot technologies [9,10]. Many methods have been proposed to introduce adhesive forces between the wall and the robot, for example negative air pressure, directional adhesive structures, magnetic force, and electrostatic force [11–15]. Liu et al. [16] discussed the electrostatic adhesion force while designing a wall climbing robot prototype. Their theoretical model considered a planar array of parallel electrodes, insulation film around the electrode panel, and a thin layer of air gap between the electrode panel and the climbing wall. The electrodes acted as the source of the electrostatic field and the air gap corresponded to the wall surface roughness. Maxwell Stress Tensor was used to calculate the electrostatic adhesion force on the wall induced by the potentials applied to the electrodes. A similar work was done by Mao et al. [17] where the electrostatic force at the interfaces of a multilayered structure involving concentric ring electrodes was derived and compared with finite element simulation data.

Another important phenomenon concerning forces induced by charges in layered dielectric is contact adhesion. When two surfaces come into contact and then separate, charge transfer can occur between them [18] and this phenomenon is known as contact charging or contact electrification. During this process, a charge distribution is developed on each surface in contact, which is typically considered as a mosaic with oppositely charged regions on the two surfaces [19]. Contact charging is a very common phenomenon and is important to many long-practiced technologies such as photocopying, electrostatic separation and laser printing. Similar phenomena have also been observed in recent nanotechnologies. For instance, Darkins et al. found that when an anionic surfactant monolayer (Sodium Dodecyl Sulfate) self-assembles on a Titania (TiO_2) surface, it organizes itself to match the charge periodicity of the surface [20]. Contact electrification has further been used to develop triboelectric nanogenerators, which are self-powered sensors that convert mechanical triggering into

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electricity by harvesting energy from the ambient environment [21–23]. The study on contact charging has a long history, with many theoretical and experimental results including full atomic simulations to study the electrostatics of laminated dielectric systems. However, some fundamental issues such as the methods of charge transfer, polarity of charged surfaces, pattern of charge distribution on the surfaces etc. are still being debated.

Because of the mosaic of complementary charges developed during contact charging, the charged surfaces show a tendency to cling to each other, a phenomenon known as contact adhesion. Wan and co-workers [24] calculated the electrostatic attraction between two charged surfaces each having a square checkerboard pattern with domains of linear dimension and finite charge density. The two surfaces were placed in vacuum, and the electrostatic adhesion was calculated for aligned or misorientated boards. Brormann et al. [25] performed experiments to evaluate the electrostatic contributions to the work of separation during detachment of micro-structured PDMS samples from a glass surface. Theoretical calculation for the adhesion force was also conducted by assuming correlated charge mosaics on the two surfaces and adopting the results of Wan et al. [24]. A small adjustment was introduced to account for the difference in the studied systems: in Wan et al., the two surfaces have zero thickness, whereas in Brormann et al. the adhesion is between two half spaces separated by an air gap. This adjustment, however, was empirical and did not result from a rigorous solution of the electrostatic problem. In addition, the different polarizability of the three media (PDMS, glass and air gap) was not taken into consideration in the force evaluation. Bai et al. [26] examined the adhesion selectivity between two infinite plates in aqueous medium, in terms of force and interaction energy. The two plates were patterned with stripes of alternating positive and negative surface charge, each with zero net charge. Based on a one-dimensional approximation, the model predicted electrostatic forces between the two surfaces, which can be attractive or repulsive depending on the arrangement of the charged stripes. An extended study was performed later by Jin et al. [27] for the same problem, but with a more rigorous two-dimensional analysis. The results demonstrated that strong adhesion can be achieved by charge complementarity between the two surfaces. In these two works, the two surfaces are separated by an electrolyte medium, which has a much higher dielectric constant than the materials on the two sides. Effect of their dielectric constants was not discussed in Ref. [26], and only lightly mentioned in Ref. [27].

The general problem relevant to contact adhesion is: what is the force between two dielectric half spaces with surface charges separated by a dielectric gap? How is the force affected by the dielectric properties of the three domains, and by the surface charge distribution? As a step towards solving such a problem, in this work, we tackle a simpler problem, namely the force between two dielectric half spaces induced by a point charge located in the dielectric gap separating them. A schematic of the problem is shown in Fig. 1. We first determine the distribution of surface force density (force per unit area) along the interfaces and then use this result to calculate the total surface force acting on each of the interfaces. Due to linear nature of the equations governing the electric potential, our approach and results can be extended into broad applications where forces induced by a distribution of charges in layered dielectrics are present.

For the remainder of the paper, detailed formulation for calculating the surface force density and total surface force on the interfaces shown in Fig. 1 is described in Section 2. Analysis of the parameters influencing the force density and the total force is conducted in Section 3. Finally, a conclusion is given in Section 4.

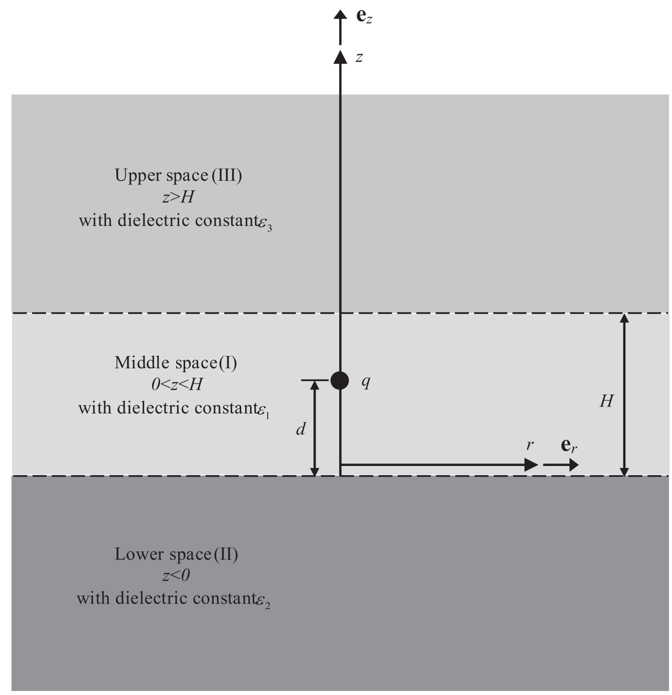

Fig. 1. A charge q at a distance d above the lower interface. The middle region is occupied by a dielectric with dielectric constant ε_1, the lower region by a dielectric with ε_2, and the upper region by a dielectric with ε_3. Regions II and III are infinitely large in the downward and upward directions respectively, whereas region I has a finite thickness of H. The dielectrics are separated by hypothetical planes that are infinitely large in the horizontal direction while having zero thickness.

2. Formulation

In Fig. 1, the space is divided into three homogeneous regions which can differ in dielectric properties: (I) located in the middle region with a thickness of H and dielectric constant ε_1, (II) located in the lower region with dielectric constant ε_2, and (III) located in the upper region with dielectric constant ε_3. All regions extend to infinity in the horizontal direction; as well region (II) extends to infinity downwards and region (III) extends to infinity upwards. A point charge q is located in region (I) at a distance of d from the lower interface. We are interested in determining the forces on the upper and lower interfaces caused by the point charge, in terms of the physical parameters q, d, H, ε_1, ε_2, and ε_3. In reality, these forces can cause deformation of the interfaces if the materials are soft, but solving the deformation field is out of the scope of this work.

To determine the surface force, we apply the following procedure. First, the electric potential ϕ in all three regions is determined by solving the Laplace equation of electrostatics with proper boundary conditions. Considering the axisymmetry of the problem, cylindrical coordinates r and z as shown in Fig. 1 are used, where the origin is located on the lower interface and the z-axis passes through the point charge. The boundary value problem for ϕ can be developed and solved using the technique of Hankel transform [28]. Knowing the electric potential throughout the space, the electric field $\mathbf{E}$ can be calculated which, in the cylindrical coordinate with axisymmetry, reads

$$\mathbf{E} = -\mathbf{e}_r \frac{\partial \phi}{\partial r} - \mathbf{e}_z \frac{\partial \phi}{\partial z}, \tag{1}$$

where $\mathbf{e}_r$ and $\mathbf{e}_z$ are the unit vectors in r and z directions respectively as shown in Fig. 1. The electric field along the angular