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A parallel 2d finite volume scheme for solving systems of balance laws with nonconservative products: Application to shallow flows

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Abstract

The goal of this paper is to construct parallel solvers for 2d hyperbolic systems of conservation laws with source terms and nonconservative products. More precisely, finite volumes solvers on nonstructured grids are considered. The method of lines is applied: at every intercell a projected Riemann problem along the normal direction is considered which is discretized by means of the numerical schemes presented in [M.J. Castro, J. Macías, C. Parés. A *Q*-scheme for a class of systems of coupled conservation laws with source term. Application to a two-layer 1-D shallow water system, ESAIM: M2AN 35 (1) (2001) 107–127]. The resulting 2d numerical schemes are explicit and first order accurate. The solver is next parallelized by a domain decomposition technique. The specific application of the scheme to one- and two-layer shallow water systems has been implemented on a PC's cluster. An efficient data structure based on OOMPI (C++ object oriented extension of MPI) has been developed to optimize the data exchange among the processors. Some numerical tests are next presented to validate the solver and the performance of its parallel implementation. Finally the two-layer shallow water model is applied to the simulation of the steady exchange through the Strait of Gibraltar.

Keywords: Parallelization; Domain decomposition; Finite volume schemes; Conservation laws; Source terms; Nonconservative products; Shallow water systems; Two-layer problems; Geophysical flows; Strait of Gibraltar

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1. Introduction

This article deals with the discretization of 2d hyperbolic systems of conservation laws with source terms and nonconservative products by means of finite volume solvers on nonstructured grids. Problems of this nature arise in computational fluid dynamics. We are concerned in particular with the simulation of free-surface waves in shallow layers of homogeneous fluids or internal waves in stratified fluids composed by two shallow layers of immiscible liquids. The motion of a layer of homogeneous fluid is supposed here to be governed by the shallow water system, formulated under the form of a conservation law with source terms or *balance law*. In the stratified case, the flow is supposed to be governed by a system composed by two coupled shallow water systems. The global system can be formulated under the form of two coupled balance laws, the coupling terms having the form of nonconservative products.

We are mainly interested in the application of these systems to geophysical flows: models based on shallow water systems are useful for the simulation of rivers, channels, dambreak problems, etc. Stratified fluids are ubiquitous in nature: they appear in atmospheric flows, ocean currents, estuarine systems, etc. This is the situation, for instance, in the Strait of Gibraltar, where surface water from the Atlantic inflows over saltier westward-flowing Mediterranean water.

It is well known that numerical methods that are suitable for solving conservation laws can fail in solving systems of balance laws, specially when approaching equilibria or near to equilibria solutions. Bermúdez and Vázquez-Cendón have shown in [29,3] that methods based on Roe's discretization of the flux terms and *upwinding* the source terms suitably solve these difficulties. For the particular case of the shallow water system, these authors introduced the condition called *conservation property* or $\mathscr{C}$-*property*: a first order scheme is said to satisfy this condition if it solves, exactly or up to the second order, the steady state solutions corresponding to water at rest. The idea of constructing numerical schemes that preserve some equilibria, which are called in general *well-balanced* schemes, has been extended in different ways: see [4] and the references therein, [18,13–15,20].

The presence of nonconservative products add more difficulties: in [12] it was shown that standard methods for conservation laws can fail to convergence when they are applied to coupled systems of conservation or balance laws. In this latter article, some stable numerical schemes for 1d coupled systems of balance laws generalizing those introduced in [29,30,3] were presented. In the particular case of the two-layer shallow water system, the well-balance property required to the numerical schemes was the natural extension of the $\mathscr{C}$-*property*: equilibria corresponding to water at rest had to be preserved. In [10] this work has been extended to the more complex system corresponding to 1d two-layer shallow water models in symmetric channels with irregular geometries.

Besides the difficulties related to source terms and nonconservative products, some specific problems appear in the particular case of shallow water systems like the numerical simulation of wet/dry fronts appearing when the thickness of the water layer vanishes. This difficulty also arises in two-layer fluids when the thickness of at least one of the layers vanishes. In real flows these situations appear in coastal simulations, upwelling events, etc. In [21,8,11] some modifications of the numerical scheme presented in [12] were proposed to handle with this difficulty.

A specific difficulty related to the two-layer shallow water system is the appearance of Kelvin–Helmholtz instabilities due to the destabilizing effect of shear, which may overcome the stabilizing effects of stratification. When these instabilities develop, the P.D.E. system loses its hyperbolic character and the numerical scheme becomes unstable due to the appearance of complex eigenvalues. In this case, the assumptions of the two-layer model considered here are no longer valid: the interface becomes a rolled-up layer which cannot be represented as the graph of a function. Nevertheless, the model can be useful to simulate flows where Kelvin–Helmholtz instabilities are expected to appear only in a sporadic and local way. In this case a correction has to be added to the numerical scheme to support instability episodes. In [18] a numerical