Accepted Manuscript

Analytical solution to and error analysis of the quaternion based similarity transformation considering measurement errors in both frames

Guobin Chang, Tianhe Xu, Qianxin Wang, Ming Liu

PII: S0263-2241(17)30399-8
DOI: http://dx.doi.org/10.1016/j.measurement.2017.06.013
Reference: MEASUR 4813

To appear in: *Measurement*

Received Date: 10 October 2016
Revised Date: 4 June 2017
Accepted Date: 12 June 2017

Please cite this article as: G. Chang, T. Xu, Q. Wang, M. Liu, Analytical solution to and error analysis of the quaternion based similarity transformation considering measurement errors in both frames, *Measurement* (2017), doi: http://dx.doi.org/10.1016/j.measurement.2017.06.013

Analytical solution to and error analysis of the quaternion based similarity transformation considering measurement errors in both frames

Guobin Chang[a,c], Tianhe Xu[b,c,]*, Qianxin Wang[a,c], Ming Liu[d]
[a] School of Environmental Science and Spatial Informatics, China University of Mining and Technology, Xuzhou 221116, China
[b] Institute of Space Science, Shandong University, Weihai 264209, China
[c] State Key Laboratory of Geo-information Engineering, Xi'an Research Institute of Surveying and Mapping, Xi'an 710054, China
[d] Key Laboratory of Aviation Information Technology in Universities of Shandong, Binzhou University, Binzhou 256603, China
* Corresponding author at School of Environmental Science and Spatial Informatics, China University of Mining and Technology, Xuzhou 221116, China.
Email address: guobinchang@hotmail.com

Abstract: The similarity transformation between two coordinate frames, is widely adopted in science and engineering. The transformation parameters are estimated using coordinate determinations of a set of common points in both frames. The quaternion is employed to represent the rotation transformation; and a 3×1 error vector is defined to represent the quaternion estimation error. Coordinate determinations in both frames are assumed noisy. An analytical least-squares solution is derived in which the quaternion estimate is the eigenvector of a 4×4 symmetric matrix corresponding to its largest eigenvalue. It is found that as long as a practically meaningful quaternion estimate exists, the largest eigenvalue must be single. Error analysis of this solution is investigated in detail in which the error analysis of the largest eigenvalue-eigenvector pair plays a pivotal role. Monte Carlo experiments are conducted and the results validate the consistency of the developed error analysis.

Key words: Similarity transformation; least-squares; quaternion; error analysis; eigen decomposition.

1 Introduction

In many engineering practices, the coordinates of a point in 3D space (expressed usually in different coordinate frames) can be determined from measurements often treated as nonlinear functionals. It is often necessary to transform the coordinates in one frame into that in another. To fulfill this, a functional model, named the coordinate transformation model, connecting the two frames, should be first determined. This is called the coordinate transformation problem [1]. There should be two tasks concerning determining the model, named determining the form of a model and estimating the parameters in the this model [2]. In this work, only the latter is investigated while the form of the model is predetermined, as the similarity transformation model, also called the Helmert model in geodesy. However the following two is noted. First, due to the practical complexities, i.e., the unknown error patterns [3], no true transformation model exists in general. One model is employed only because it is useful, but not because it is true. Besides the similarity model, there are many others in the literature, often more complicated. Second, the model can be selected or validated using measurements, through e.g., cross validation [4], hypothesis test [5], or model selection theory [6].

In the predetermined similarity model, three different sub-transformations are included, named the translation, the rotation, and the isotropic scale. It is natural to represent the translation and the scale as a 3×1 vector and a positive scalar, respectively. However, the case for the rotation is a little more