Measurement 100 (2017) 139–149

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

A CAD-FEM-QSA integration technique for determining the time-varying meshing stiffness of gear pairs

Jiaxing Zhan *, Mohammad Fard, Reza Jazar

School of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, Melbourne 3001, Australia

ARTICLE INFO

Article history:
Received 3 August 2015
Received in revised form 2 November 2016
Accepted 28 December 2016
Available online 31 December 2016

Keywords:
CAD
FEM
Quasi-static algorithm
Time-varying
Meshing stiffness
Gear

ABSTRACT

Estimation of time-varying meshing stiffness (TVMS) is a vital process as it is one of the primary sources of vibration and noise. There are two ways to evaluate the TVMS, the analytical method (AM) and the finite element method (FEM). Owing to the complex geometries and the use of empirical values, analytical model is not precise to perform the exact calculation. The current FEM is inconvenient due to the use of complex programming codes. Therefore, the first aim of this study is to develop a new technique to determine the TVMS based on NX, ANSYS Workbench, and Quasi-static Algorithm (QSA). The second aim is to compare results of the analytical method (AM) and the proposed method. The effects of tip radius and misalignment of gear pair are also analysed using this new method.

1. Introduction

As a periodic function caused by the change in the number of contact tooth pairs and the contact positions of the gear teeth, gear stiffness associated with elastic tooth bending varies periodically with time. It is one of the primary sources of vibration and noise, causing severe vibrations and instabilities under certain operating conditions [1,2]. Thus it is vital to understand the time-varying meshing stiffness (TVMS) of gears.

To date, considerable literature has focused on the study of TVMS using analytical method (AM). Yang and Lin [3] reported the total mesh stiffness of a pair of meshing gears as a function of the rotation angle of the gear using the potential energy method by considering Hertzian, bending and axial compressive energy. Tian [4] refined this method by taking the shear mesh stiffness into consideration. Shen et al. [5] analysed the gear mesh stiffness based on incremental harmonic balance method. Fernandez et al. [6] described an advanced model for the analysis of gear meshing stiffness. The obtained results of TVMS shown good agreement compared with the model of Cai [7]. Pedersen and Jorgensen [8] found that the stiffness of an individual tooth can be expressed in a linear form by assuming that the contact width is constant.

To prevent potential failures in gear transmission systems, researchers have developed a large number of analytical models in predicting gear tooth crack propagation and diagnosing gear fault growth. Chaari et al. [9,10] quantified the gear mesh stiffness reduction due to the tooth crack, tooth breakage and spalling by considering the bending, fillet-foundation and contact deflections. Chen and Shao [11] claimed that the crack along its depth direction should be straight but with a non-uniform distribution along tooth width. Their model makes it possible to check the effectiveness of algorithms in fault diagnosis and condition monitoring, especially for the crack at early stage. They also used their model to observe the dynamic response of planetary gear sets with different crack sizes [12]. In the study of Mohammed et al. [13], instead of using the method in [11], they evaluated the gear mesh stiffness by using parabolic curves to deal with the tooth thickness reduction. Their model was improved in [14] by considering the misalignment of root circle and base circle due to the change of number of teeth. Later, a similar algorithm was also proposed by Wan et al. [15]. Compared with ISO standard 6336-1-2006 [16], the results obtained with their method were more accurate than that of calculated by Wu et al. [17].

Many researchers also investigated the TVMS of gears using finite element (FE) models. Sirichai [18] first investigated the torsional mesh stiffness of gears in 1999. Then his work was refined by Wang and Howard [19,20]. Chaari et al. [9] analysed the TVMS of one tooth without considering the contact between pinion and wheel tooth pairs in order to simplify the computation of tooth deflection. A similar work was reported in [13] by utilizing a 2D FE model to estimate the TVMS. Li conducted loaded tooth contact

* Corresponding author.
E-mail address: raulzjx@hotmail.com (J. Zhan).

http://dx.doi.org/10.1016/j.measurement.2016.12.056
0263-2241/

analysis, deformation and stress calculations of spur gears with different addendums and contact ratios in [21]. The TVMS of gears was analysed by a hybrid method using face-contact model of teeth, mathematical programming method and 3D finite element method (FEM).

In the above-mentioned literature, there are two ways to evaluate the TVMS of gears, the AM and the FEM. The AM has a higher computational efficiency but the bigger disadvantage of AM is inaccuracy. In other words, AM is not precise to perform the exact calculation due to the complex geometries of gears. The calculated stiffness may also cause errors in the response spectrum due to the introduction of empirical values. On the other hand, the calculation result of FEM is closer to the real situation and more reliable because it can consider accurate deformations of involute, transition curve and fillet foundation [13,14]. However, to measure the TVMS, previous work was mainly based on single point analysis of mesh stiffness. The analysis of successive gear meshing positions requires repeating the setup process. Although scholars developed various types of programme codes to save the computation time, the analysis of the TVMS by FEM is still time-consuming. Furthermore, it is difficult for an inexperienced researcher without sufficient ability in the use of computer language. All in all, as concluded in [22], previous studies mostly focused on 2D analysis of gears pairs. The 3D analysis method is relatively insufficient.

In this paper, instead of focusing on complex programming codes, the TVMS corresponding to the rotation position of the gear pair is investigated based on NX, ANSYS Workbench, and Quasi-static Algorithm (QSA). Our attempt is to develop an innovative CAD-FEM-QSA integrated approach for determining the TVMS, which incorporated the advantages of both AM and FEM. Furthermore, another aim of this study is to compare results of the analytical method based on Cai's approach and ISO standard. Effects of tip radius and the misalignment of gear pair are also analysed using proposed method.

2. Mathematical model

2.1. Analytical model of the TVMS

According to method of Cai [7], the normalised linear stiffness with respect to the mean value can be expressed as follow:

$$k(t) = \frac{1}{0.85\varepsilon}\left[\frac{-1.8}{(\varepsilon t_z)^2}t^2 + \frac{1.8}{\varepsilon t_z}t + 0.55\right] \tag{1}$$

where t_z is the meshing period or normal pitch length of the gear pair, ε is the contact ratio and t is the instant considered.

According to ISO 6336, for spur gears with $\varepsilon \geqslant 1.2$ and helical gears with $\beta \leqslant 30^\circ$ the mesh stiffness is:

$$c_\gamma = c'(0.75\varepsilon + 0.25); \tag{2}$$

where the maximum stiffness c' can be obtained from

$$c' = c'_{th} C_M C_R C_B \cos\beta; \tag{3}$$

where c'_{th} is the theoretical single stiffness, C_M is the correction factor which accounts for the difference between the measured values and theoretical values, C_M is the gear blank factor that accounts for the flexibility of gear rims and webs, C_B is the basic rack factor accounts for the deviation of the actual basic rack profile of gear, from the standard basic rack profile, β is the helical angle at reference pitch diameter. On the other hand, c'_{th} can be calculated as:

$$c'_{th} = \frac{1}{q'} \tag{4}$$

where q' is the minimum value for the flexibility of a pair of teeth;

$$q' = C_1 + \frac{C_2}{z_{n1}} + \frac{C_3}{z_{n2}} + C_4 x_1 + \frac{C_5 x_1}{z_{n1}} + C_6 x_2 + \frac{C_7 x_2}{z_{n2}} + C_8 x_1^2 + C_9 x_2^2 \tag{5}$$

The values of $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ are given in Table 1.

Then the linear-translation tooth meshing stiffness along the line of contact, K_l, can be calculated by

$$K_l = k(t) c_\gamma B \tag{6}$$

where B represents the tooth width of gear pairs.

2.2. Quasi-static algorithm

For the sake of clarity, a two-DOF spur gear pair is used in this paper (Fig.2), where shafts and bearing are assumed to be rigid. It is also assumed that the gear pair is in perfect mesh condition. Thus, the gear mesh inaccuracies, such as gear eccentricity, unequal tooth width or pitch deviations are negligible in this study.

Based on the above consideration and the D'Alembert's principle, the system equation of this classical model with respect to the angular rotations φ_p and φ_g of the pinion and gear, respectively, can be written as [23]:

$$I_p\ddot{\varphi}_p + C(\dot{\varphi}_p - \dot{\varphi}_g - \dot{e}) + K_t(\varphi_p - \varphi_g - e) = T_p \tag{7}$$

$$I_g\ddot{\varphi}_g - C(\dot{\varphi}_p - \dot{\varphi}_g - \dot{e}) - K_t(\varphi_p - \varphi_g - e) = -T_g \tag{8}$$

where I_p and I_g represent the rotary inertia for pinion and gear, respectively, and C and K_t are the damping coefficient and torsional meshing stiffness of the system, respectively. The angular velocity is represented by $\dot{\varphi}_p$ and $\dot{\varphi}_g$ while $\ddot{\varphi}_p$ and $\ddot{\varphi}_g$ are the angular acceleration. T_p and T_g denote the external torques acting in the system. The quantity 2e is defined as the total gear backlash.

When pinion and gear revolute with very slow speed rate, the inertial effects produced in gear pair are very small and can be ignored. Therefore, the quasi-static equations can be obtained by deleting derivatives terms in Eqs. (7) and (8):

$$T_p = K_t(\varphi_p - \varphi_g - e) \tag{9}$$

$$T_g = -K_t(\varphi_p - \varphi_g - e) \tag{10}$$

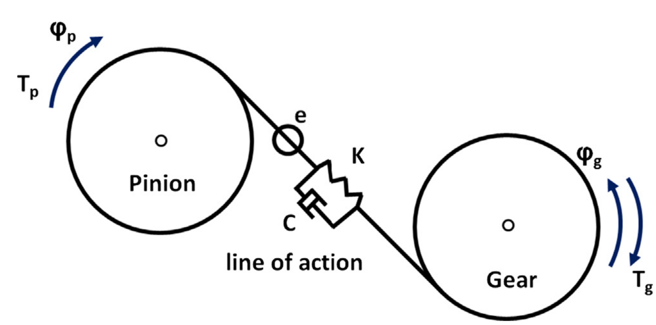

Fig. 1. Torsional model of gear pair.

Table 1
Coefficients used in Eq. (5).

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
0.04723	0.15551	0.25791	−0.00635	−0.11654	−0.00193	−0.24188	0.00529	0.00182