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An application of box counting method for measuring phase fraction

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ABSTRACT

Box counting is a simple method to calculate the fractal dimension of an image, but it is poorly understood for estimating phase fraction from the microstructure image. Here, we investigate six recipes to estimate the phase fraction by box counting method (BCM). The optimal recipe is determined by applications in the microstructures of Ni-B alloys, and is further verified by theoretical analysis. As compared to pre-existing methods, BCM is more convenient and can provide accurate phase fraction by lower cost.

1. Introduction

Phase fraction is an important factor to determine the material properties [1]. Many technologies and methods have been applied to calculate the phase fractions, for instance, calorimetric principle method [2], cooling curve baseline analysis (CCBA) [3,4], computer-aided cooling curve analysis (CA-CCA) [5–7], differential thermal analysis (DSC) [8,9], in situ X-ray radiographic observation [10,11] and microstructure imaged pixel analysis [3,12]. The CCBA and CA-CCA methods have certain errors since the heat release fraction does not completely coincide with the phase fraction. DSC measurement is limited in the small sample, and always influenced by error of heat release. In situ X-ray radiographic analysis is too expensive for phase fraction measurement. Imaged pixel method is an easy method, but only available for the sample well etched (the phase color is uniform). Besides these methods, the box counting method (BCM) can also be used for measuring the phase fraction.

Box counting is a classical method for estimating the fractal dimension of a structure [13–17]. In this method the image is covered with an array of square boxes, each box of size L and the number of those boxes N that cover the fractal pattern is counted. The relationship between N and the size of the box can be represented by $N \sim L^{-d}$, where d is the box-counting dimension (fractal dimension) and is a function of the geometry of the pattern. Plotting the negative log of N against the log of L produces a curve whose slope estimates d [13]. Using box counting (BCM), Kruelle et al. studied fractal structure in chaotic billiards [14]; Newbury et al. studied the fractal behavior in the magnetoresistance of Chaotic Billiards [15]; Ree studied the chaotic dynamics fractal analysis on a closed classical hard-wall billiard [16]; Városi studied the spectrum of fractal dimensions of passively convected scalar gradients in chaotic fluid flows [17]. For measuring phase fraction from a microstructure image, BCM has its own benefits such as convenience and flexibility even when the imaged pixel analysis cannot be successfully performed. However, up to now, BCM for determining phase fraction measurement, especially the calculation recipes have not been reported, and the error analyses of BCM has not been revealed yet.

In this study, the BCM for measuring phase fraction was studied upon the microstructures of Ni-B alloys. Six calculation recipes were performed, and the errors of these recipes were discussed.

2. Method descriptions

A microstructure image (680 × 512 pixels) of hypereutectic Ni-4.5 wt.%B alloy including two phases is shown in Fig. 1a. The fractions of Ni_3B phase (yellow) and the lamellar eutectic need to be measured. Since the lamellar eutectic consists of α-Ni and Ni_3B phases, there are no obvious differences in color between the lamellar eutectic and Ni_3B grain phases. Thus, the fractions of Ni_3B grains and the lamellar eutectic cannot be directly measured by imaged pixel method. If this method must be used, the lamellar eutectic should be re-colored carefully at first, instance as shown in Fig. 1b, the lamellar is colored as black, and then the phase fraction of Ni_3B grains can be measured as 43.1%. The measurement by imaged pixel method is so inconvenient that it is not suitable for the case where the color of phase image is non-uniform or the sample is not well etched.

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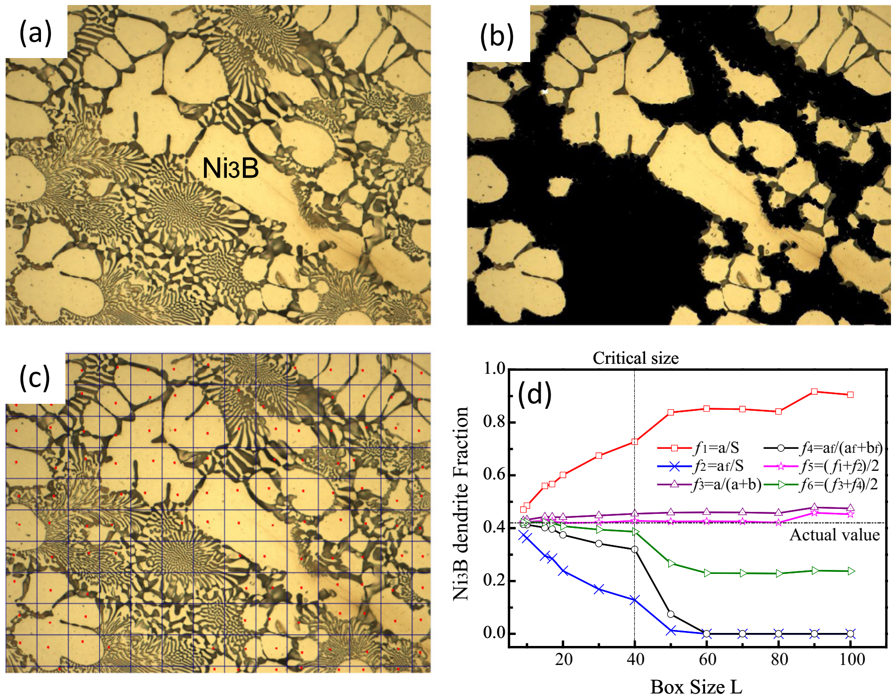

Fig. 1. Microstructure of hypereutectic Ni-4.5 wt.% B alloy and the measured phase fractions of Ni_3B grains: (a) original image (680 × 512 pixels); (b) coloring the lamellar as black for imaged pixel measurement; (c) the image divided by grids with 50 × 50; (d) the results from the six recipes.

The problems can be solved by using BCM. For this method, the image is covered by the boxes with side length $L \times L$ at first then the number of the boxes containing Ni_3B phase (including some boxes full of Ni_3B phase and the others partially filled with Ni_3B phase) is counted as a, as shown in Fig. 1b. If the total number of the boxes is determined as S according to the sizes of the image and the grids, the fraction of Ni_3B phase can be estimated by,

$$\text{Method 1}: \; f_1 = a/S \tag{1}$$

If the number of the boxes only contains Ni_3B; then, the grain phase is counted as a_f, the fraction of the Ni_3B grains can also be estimated by,

$$\text{Method 2}: \; f_2 = a_f/S \tag{2}$$

Furthermore, if the number of the boxes containing the lamellar eutectic is counted as b, the fraction of the Ni_3B grains can thus be estimated by,

$$\text{Method 3}: \; f_3 = a/(a+b) \tag{3}$$

And if the number of the boxes only containing lamellar eutectic is counted as b_f, the fraction of the Ni_3B grains can also be estimated by,

$$\text{Method 4}: \; f_4 = a_f/(a_f+b_f) \tag{4}$$

Undoubtedly, the results from methods 1 and 3 are larger than the accurate value, while those from methods 2 and 4 are smaller than the accurate value. So the average values of methods 1, 2 and methods 3, 4 may give more accurate results, i.e.

$$\text{Method 5}: \; f_5 = (f_1+f_2)/2 = (a+a_f)/2S \tag{5}$$

$$\text{Method 6}: \; f_6 = (f_3+f_4)/2 = [a/(a+b)+a_f/(a_f+b_f)]/2 \tag{6}$$

Eqs. (1)–(6) are six calculation recipes for estimating the phase fraction from an image by BCM. Defining the number of the boundary boxes including both phase A (Ni_3B grains) and phase B(lamellar eutectic) as I_{ab}, then there are several relationships,

$$a+b-S = I_{ab} \tag{7}$$

$$a_f+b = S \tag{8}$$

$$a+b_f = S \tag{9}$$

$$I_{ab} = a-a_f = b-b_f \tag{10}$$

where b_f is the number of the boxes containing only phase B (lamellar eutectic).

Using boxes with sides length of L = 9–100, the microstructure image of Ni-4.5 wt.%B alloy is covered, and the numbers of the boxes containing Ni_3B phase are counted, as shown in Fig. 1c. According to Eqs. (1)–(6), the calculated fractions of Ni_3B phase are as listed in Table 1. The relations between the box sizes and the calculated results are plotted in Fig. 1d. It shows that, when the box size is small enough, the calculated fractions of Ni_3B phase from BCM (Fig. 1d) are much closed to that from the imaged pixel method (43.1%). It is important to note that, the results from methods 3 and 5 are more accurate than those from methods 1, 2, 4 and 6 for any sizes of box size, which means that methods 3 and 5 may eliminate some errors. In addition, in Fig. 1d, a critical point (where the slope changes suddenly) appears at L = 30–50 for the curves of f_1 and f_2; and if $L < 30$, the values of f_1 and f_2 will quickly approach to the actual value with decreasing of L, where the actual value is the exact value upon the box size closed to zero. Interestingly, the average size of Ni_3B grains is also between 30 and 50, it implies that critical point depends on the phase (grain) size.

In order to test BCM with another example, the microstructure image (1200 × 904 pixels) of hypoeutectic Ni-3.3 wt.%B alloy, which consists of primary α-Ni phase (dark) and lamellar eutectic, was studied (Fig. 2a). Due to the non-uniform color of α-Ni grains, the phase fraction cannot be measured by the imaged pixel method. However, it can be measured directly by BCM. Dividing the image with the grids of L = 9–100, the box numbers are counted as listed in Table 2. Then from Eqs. (1)–(6), the fractions of α-Ni grains can be calculated. The relations between the calculated results and the box sizes are plotted in Fig. 2b. It can be found that the smaller the box size is, the more accurate phase fraction will be obtained; for any size grids; the results from methods 3