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Educational aspects of uncertainty calculation with software tools

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ABSTRACT

Despite its importance, uncertainty of measurements and parameters is frequently neglected by practitioners in the design of systems even in safety critical applications. Thus, problems arising from uncertainty may only be identified late in the design process or even remain. This can lead to additional costs and increased risks. Although there exists numerous tools to support uncertainty calculation, reasons for limited usage in early design phases may be low awareness of the existence of the tools and insufficient training in the practical application.

In order to enhance the widespread use of such tool support we suggest a teaching concept for uncertainty calculation in measurement science education that is directly based on the utilization of software tools. Although the developed material is currently based on the GUM (Guide to the expression of uncertainty in measurement) method we believe that it is also useful with other methods. Additionally, the concept goes beyond the scope of measurement uncertainty quantification demonstrating that it is also useful for system analysis and optimization.

1. Introduction and motivation

The fact that measurement results are more than just numeric values is well known and accepted when it comes to physicals units. Thus, it is common practice to report the unit together with the numeric result of a measurement. However, it is not as common to emphasize that measurement results are usually composed by realizations of random variables. The ideal way to represent random variables is to provide the probability distribution. However, this may be difficult, impractical or even impossible in many situations. As an alternative, the uncertainty attributed to the measurement result may be reported in terms of certain parameters of the probability density function. In the simplest case we could just report a single additional parameter to indicate if the distribution is narrow or wide; i.e. a non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used [1]. In other words, it provides information about the remaining uncertainty about the measurand [2]. The idea to develop a new guide for the treatment of uncertainty in measurement was to overcome some of the limitations that are associated with the previously used term error [3] and led to the change in the treatment of measurement uncertainty from an Error Approach (sometimes called Traditional Approach or True Value Approach) to an Uncertainty Approach [1]. With respect to metrology, the uncertainty reflects the fact that measurements can only provide incomplete knowledge and that a measurement is only useful when the lack of knowledge is somehow quantified.

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This is particularly true with respect to safety and reliability. Consider, for example, a monitoring system that should validate that a certain parameter lies within a certain interval. If the measurement uncertainty of the monitoring system becomes larger than the interval to be monitored, then the monitoring system can *never* be used to validate that the parameter is actually *within* the interval; it can only be used to validate that the parameter (with high probability) resides *outside* of the interval. This may not be apparent for a user or even for a developer of such a system, in particular considering that the engineer may not be an expert in stochastics and uncertainty quantification. Therefore, it seems to be reasonable to provide a method that is commonly accepted by practitioners and experts, can easily be applied for a wide range of problems and still provides good results (even if they may not be optimal in a theoretical sense).

In 1977, as it was recognized the existence of a lack of international consensus on the expression of uncertainty in measurement, the world's highest authority in metrology, the Comité International des Poids et Mesures (CIPM), requested the Bureau International des Poids et Mesures (BIPM) to address the problem in conjunction with the national standards laboratories and to make a recommendation. The effort finally led to the development of the Guide to the Expression of Uncertainty in Measurement (GUM) [3]. According to the GUM, the ideal method should be universal (applicable to all kinds of measurements and to all types of input data used in measurements), internally consistent (directly derivable from the components that contribute to it), and transferable (possibility to directly use the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used).

With respect to one of the initial requirements for such a recommendation – i.e. the approach has to be universal – the GUM [3] treats all uncertainty contributions identically, more or less as if the distributions were Gaussian and the relations were linear. The Central Limit Theorem is significant in this context because it shows the very important role played by the variances of the input quantities' probability distributions, compared with that played by the higher moments of the distributions, in determining the form of the resulting convolved distribution of Y. Further, it implies that the convolved distribution converges towards the normal distribution even for comparatively small numbers of contributing parameters. For instance, the convolution of as few as three rectangular distributions of equal width is approximately normal [3].

However, the GUM working group was aware that there are limitations of the GUM [3] method and in supplements [3–5] suggested to use Monte Carlo sampling in certain cases. A recent survey [6] on current research activities in the field of measurement uncertainty reports that most recent work addresses the GUM. Consequently, the present paper focuses on this approach, which has a wide acceptance within the field of metrology. Simplicity of tools that implement the method is also crucial for the acceptance, as stated e.g. in [7]. Similarly, the authors of [8] emphasize the beneficial role that tools may play to eventually make uncertainty propagation an inherent component of computational procedures instead of an optional addendum. With the same motivation, we aim to bring students in touch with such tools early in their curriculum.

Our approach uses a tool that integrates well into a mathematical programming environment with which our students are familiar. We currently use Matlab, but the approach may also be used with other environments, e.g. [9] for students well trained in Java. The basic educational concept was presented in [10]. In the present paper, we discuss additional aspects such as the numeric representation of uncertainty and the utilization of the concept beyond the scope of classical measurement uncertainty quantification.

Our educational concept is directly applicable to two toolboxes for Matlab [11]; i.e. Metas.UncLib MatLab toolbox [12] and a toolbox developed by our group. Both toolboxes include an implementation of the GUM tree method [13]/automatic differentiation [14]. The toolboxes are similar in basic usage and basic functionality. Differences mainly relate to reporting of uncertainty and analysis of uncertainty contributions (in part as a response on student feedbacks). Furthermore, to keep things transparent for the students they can have a look into the MatLab source code rather than obtaining a "black box".

It should be noted that there have been many discussions about the GUM and several alternative approaches exist as discussed e.g. in [15,16] and recently in [17]. Additionally, a revision of the GUM [18,19] is in preparation. However, the teaching concept that we present in this paper can be used with different approaches as long as it is possible to implement them in an automatic tool. In this context it will be important to outline the methods and explain their advantages and disadvantages to the students. However, the main objective is to sensitize students for the concept of uncertainty such that it becomes a part of practices of daily life. This can be achieved with different approaches and we currently use the classical GUM approach.

2. Software tool concept

2.1. Assigning and reporting uncertainty

In principle, the GUM [3] has two different types of uncertainty evaluation. The Type A evaluation uses statistical methods, i.e. the uncertainties are obtained from experiments by drawing samples from the distribution and calculate the standard uncertainty based on the empirical data. In the Type B evaluation, the uncertainty of input quantities is known a priori. In order to obtain the combined standard uncertainty attributed to the final measurement result it is necessary to determine the individual contributions of the input quantities.

In this paper, we focus on the determination of the combined standard uncertainty based on the standard uncertainty of the input quantities. Our examples are based on Type B uncertainties, which represents a common case where the prior knowledge is provided by the manufactures of the devices, e.g. instrumental measurement uncertainty for voltmeters or sensors in the respective

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