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New methods to estimate the observed noise variance for an ARMA model

Kedong Wang [a,*], Yuxia Wu [a], Yifeng Gao [a], Yong Li [b]

[a] School of Astronautics, Beihang University, Beijing 100191, China
[b] School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia

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ABSTRACT

For an ARMA model with an observed noise, the observed noise variance estimation is not only a part of the model identification, but also its estimation accuracy affects the following MA parameter estimation accuracy directly. However, it is difficult to improve the estimation accuracy of the observed noise variance, especially when the observed noise variance is small. In this paper, two new methods are proposed to estimate the observed noise variance accurately. In the first method, the lower lags of the autocovariance function are used to estimate the observed noise variance with high estimation accuracy, but it is valid only when the AR order is greater than the MA order. In the second method, the ARMA model is approximated as a high-order AR model so that it is effective even though the AR order is equal to or less than the MA order. If the observed noise variance is too small, its estimation error may be too large to valid the estimate. An empirical criterion is proposed to judge the necessity of estimating the observed noise variance. The proposed methods are verified by simulations and applied to the random noise modeling for gyroscopes tentatively.

1. Introduction

Autoregressive moving average (ARMA) has been widely applied to model the colored noise of sensors, the time series of economic data, and other data processing applications [1–5]. An ARMA model will degenerate into an autoregressive (AR) or a moving average (MA) one if its MA or its AR parameters are zeros. Although many efficient methods have been proposed to estimate the parameters of a noise-free ARMA model, they are invalid if the measurements are corrupted by noise [6]. Hence, more efforts have been devoted to propose the effective methods for an ARMA model with an observed noise [6–15]. To mitigate impact of the observed noise on the identification accuracy of the ARMA model parameters, the noise-compensated modified Yule-Walker (NCMYW) equations have been proposed to estimate the AR parameters and the observed noise variance simultaneously [8,9]. The MA parameters can be estimated from the residuals of the observed signal filtered by the estimated AR polynomial. The typical MA estimation methods include the maximum likelihood, the spectral factorization, and the over-parameterized signal or covariance [8,16,17]. It has been verified that the AR parameters can be estimated with the high accuracy by the NCMYW method or others [9–13]. However, it is difficult to estimate the observed noise variance accurately, in particularly when the observed noise variance is small. The observed noise variance with the degraded estimation accuracy will reduce the MA parameter estimation accuracy directly [8]. Hence, new methods are needed to improve the observed noise variance accuracy if possible. Moreover, the NCMYW method or the similar methods are effective only when the AR order is larger than the MA order. When the AR order is equal to or less than the MA order, new methods are needed to estimate the observed noise variance too.

The observed noise variance estimation is a kind of blind noise level estimation which has been studied in many fields except the ARMA modeling for the one-dimension time series of data. For instance, noise variance estimation algorithms based on wavelet transform, discrete cosine transform, principal component analysis, etc. have been developed in the past two decades for the image processing [18,19]. However, the noise-free image is unnecessary to model usually while not only the observed noise variance but also the ARMA parameters should be estimated in modeling for the one-dimension time series of data.

In this paper, the ARMA modeling is investigated for a sensor noise. The output governed by the modeled ARMA is usually different from the sensor noise and the difference is the observed noise

* Corresponding author.
E-mail address: wangkd@buaa.edu.cn (K. Wang).

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herein. Two new methods are proposed to estimate the observed noise variance accurately for the ARMA model. In the first method, it is assumed that the AR order is larger than the MA order. The estimated AR polynomial is used to estimate the observed noise variance with the low order Yule-Walker equations. Since it is relatively easier to estimate the AR parameters accurately, the observed noise variance can be accurately estimated in the first method. It is verified by the succeeding simulations that the observed noise variance can be estimated more accurately with the first method even when the observed noise variance is small. In the second method, the ARMA model is approximated by a high-order AR model to estimate the observed noise variance. Since the observed noise variance estimation is based on the approximate AR model, it should be no matter what order of AR or MA is in the second method. Therefore, the second method can be used to estimate the observed noise variance whatever the AR order is larger, equal to, or less than the MA order.

This paper is organized as follows. The two methods are described in detail in Section 2. In Section 3, the two methods are verified by Monte Carlo simulations. The estimation results with the two proposed methods are compared with the typical existing methods. The necessity to estimate the observed noise variance is also discussed. In Section 4, the two methods are verified by experiments to compensate for the gyroscope random noise. Finally, the concluding remarks are provided in Section 5.

2. Methodology

2.1. The AR-based method

A casual, stationary, and invertible ARMA(p,q) model can be written as

$$\sum_{k=0}^{p} a_k x(n-k) = \sum_{l=0}^{q} b_l e(n-l) \tag{1}$$

where $e(n)$ and $x(n)$ are the excitation and the response of the ARMA system respectively and $e(n)$ is a stationary Gaussian white noise with zero mean and the variance of σ_e^2; a_k and b_l are the AR and the MA parameters respectively and $a_0 = b_0 = 1$; p and q are the AR and the MA orders respectively, which are assumed to be known in this paper. In this subsection, it is assumed that $p > q$. Let $\boldsymbol{\theta} = [1 \quad a_1 \quad \cdots \quad a_p]^{\mathrm{T}}$ and $\boldsymbol{\varphi} = [1 \quad b_1 \quad \cdots \quad b_q]^{\mathrm{T}}$. Upon the observation of $x(n)$, $y(n)$ is corrupted by a stationary additive Gaussian white noise $v(n)$ with zero mean and variance of σ_v^2, i.e.

$$y(n) = x(n) + v(n) \tag{2}$$

where $v(n)$ is uncorrelated with $e(n)$. There is the relation

$$r_y(\tau) = \begin{cases} r_x(\tau) + \sigma_v^2 & \tau = 0 \\ r_x(\tau) & \tau \neq 0 \end{cases} \tag{3}$$

where $r_x(\tau)$ and $r_y(\tau)$ are the auto-covariance functions (ACF) of $x(n)$ and $y(n)$ respectively. According to Eq. (1), the modified Yule-Walker (MYW) equations can be formulated as

$$\begin{bmatrix} r_x(q) & r_x(q-1) & \cdots & r_x(q-p+1) \\ r_x(q+1) & r_x(q) & \cdots & r_x(q-p+2) \\ \vdots & \vdots & \vdots & \vdots \\ r_x(p+s-1) & r_x(p+s-2) & \cdots & r_x(s) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = - \begin{bmatrix} r_x(q+1) \\ r_x(q+2) \\ \vdots \\ r_x(p+s) \end{bmatrix} \tag{4}$$

where s governs the number of the equations in Eq. (4) and $s \geqslant q$. Substitute Eq. (3) into Eq. (4) and yield

$$\begin{bmatrix} r_y(q+1) & \cdots & r_y(0) - \sigma_v^2 & \cdots & r_y(q+1-p) \\ r_y(q+2) & \cdots & r_y(1) & \cdots & r_y(q+2-p) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_y(p) & \cdots & r_y(p-q-1) & \cdots & r_y(0) - \sigma_v^2 \\ r_y(p+1) & \cdots & r_y(p-q) & \cdots & r_y(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_y(p+s) & \cdots & r_y(p+s-q-1) & \cdots & r_y(s) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = \mathbf{0} \tag{5}$$

Eq. (5) is the so-called NCMYW equations. The AR parameters can be estimated by the following two approaches. The first is based on the so-called extended modified Yule-Walker (EMYW) equations. The EMYW equations are composed of the equations from the order of $p+1$ to the order of $p+s$ in Eq. (5) and $s \geqslant p$. Hence, the EMYW equations are the NCMYW equations excluding the former $p-q$ equations in Eq. (5). The EMYW equations can be solved by the least squares (LS) algorithm to estimate the AR parameters. The second method solves the NCMYW equations directly by the generalized eigenvalue problem approach to estimate the AR parameters and the observed noise variance σ_v^2 simultaneously. Since the lower order equations are included in the NCMYW equations, the AR parameter estimation accuracy of the second method is slightly higher than that of the first method. The accuracy difference between two methods is negligible as verified in the succeeding simulations.

With the assumption that $p > q$ in this subsection, the $p-q$ equations in Eq. (5) (from $q+1$ to p) are extracted from Eq. (5). It yields

$$\mathbf{R}_y \theta = \sigma_v^2 \mathbf{a} \tag{6}$$

where $\mathbf{a} = [a_{q+1} \cdots a_p]^{\mathrm{T}}$; and

$$\mathbf{R}_y = \begin{bmatrix} r_y(q+1) & r_y(q) & \cdots & r_y(0) & \cdots & r_y(p-q-1) \\ r_y(q+2) & r_y(q+1) & \cdots & r_y(1) & \cdots & r_y(p-q-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_y(p) & r_y(p-1) & \cdots & r_y(p-q-1) & \cdots & r_y(0) \end{bmatrix}$$

If the AR parameters have been estimated by the EMYW equations or the NCMYW equations, the observed noise variance σ_v^2 can be estimated as

$$\hat{\sigma}_v^2 = (\hat{\mathbf{a}}\hat{\mathbf{a}}^{\mathrm{T}})^{-1} \hat{\mathbf{a}}^{\mathrm{T}} \mathbf{R}_y \hat{\theta} \tag{7}$$

where $\hat{\mathbf{a}}$ and $\hat{\theta}$ are the estimation of $\mathbf{a}$ and θ respectively. In Eq. (7), the estimation accuracy of σ_v^2 is determined by $\hat{\theta}$ and $\mathbf{R}_y$. Since only the lower order equations in Eq. (5) are used to estimate σ_v^2, the lower lags of the ACF are included in $\mathbf{R}_y$, which can help to improve the estimation accuracy of σ_v^2. It is also noticed that the estimation accuracy of the AR parameters is relatively high. Hence, it can be expected of high estimation accuracy of σ_v^2, which will be verified in the succeeding simulations. The estimation above is called the AR-based (ARB) method hereinafter.

2.2. The equivalent AR method

Although the ARB method is expected of high estimation accuracy of σ_v^2, the method is invalid if $p \leqslant q$. In fact, the existing methods, such as the EMYW method and the NCMYW method, are also invalid if $p \leqslant q$, which is an undesirable restriction to estimate σ_v^2 in practice. In this subsection, a new method is proposed to estimate the observed noise variance accurately in arbitrary orders of the ARMA model whether $p \leqslant q$ or $p > q$.